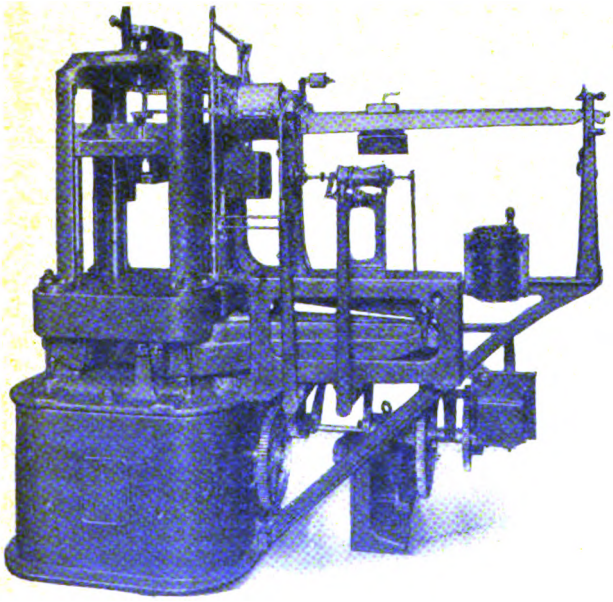
STRENGTH OF MATERIALS

PRINCIPLES OF THE THEORY—METHODS FOR CALCULATING STRENGTH AND DETERMINING DIMENSIONS OF MACHINE PARTS

BY ERIK OBERG



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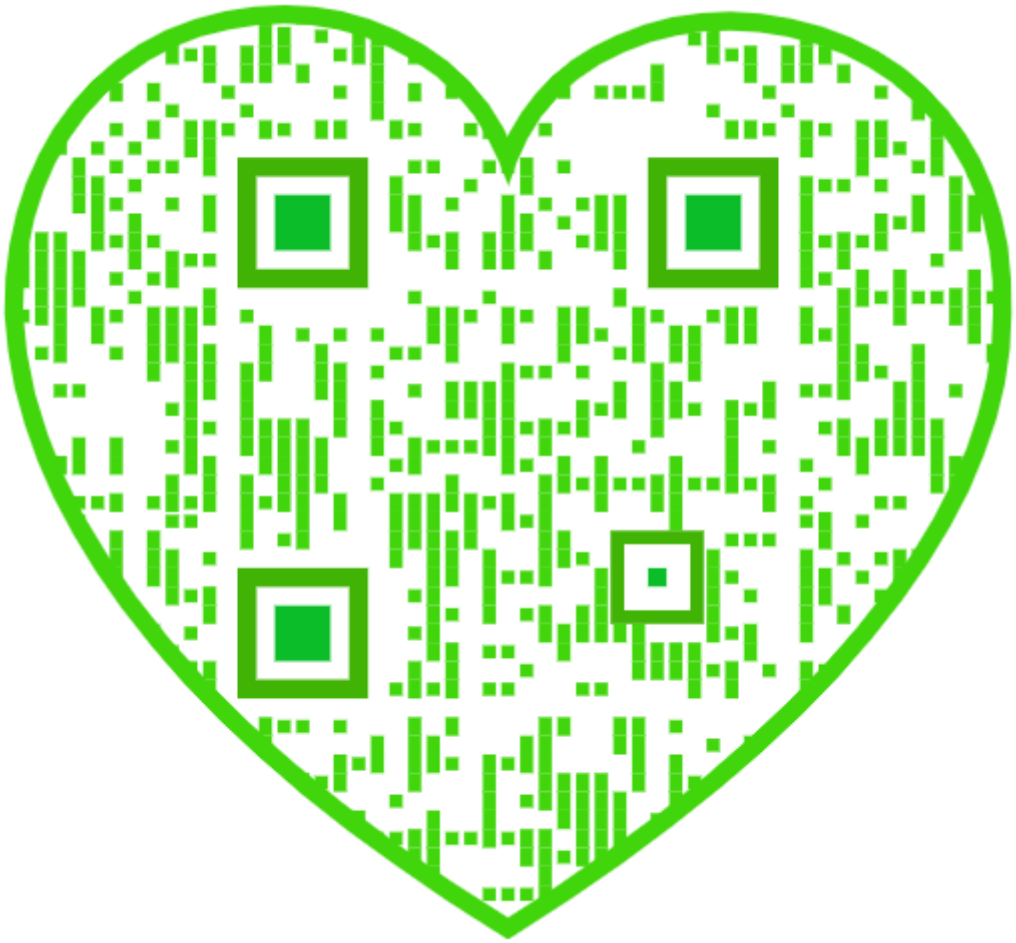

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STRENGTH OF MATERIALS

BY ERIK OBERG

CONTENTS

Principles of the Strength of Materials - - - - 3
Tension, Compression and Shearing Stresses - - - 11
Beams and Bending Stresses - - - - - - - - 18
Torsional Stresses—Shafting and Springs - - - - 30
Miscellaneous Applications - - - - - - - - 40

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PREFACE

In the design of machinery, there is nothing more important than to be able to determine the stresses to which a machine member is subjected and to give it the adequate strength for the purpose for which it is intended. It is the object of this book to give in as simple a manner as possible the principles of the methods used in calculating the strength of parts met with in machine design and engineering work generally, and to give the formulas used in such a shape that they can be directly employed by the practical man. No attempt has been made to show how the formulas are derived mathematically, as this would be impossible in a book of limited size. However, the book will be all the more acceptable to the great number of practical men, because it avoids involved mathematical treatment and presents every formula in its simplest shape. Numerous examples have also been given to show the application of the rules and formulas. While thus the subject is treated in as simple a manner as possible, the book is still comprehensive and covers all the more important questions connected with the subject.

A special effort has been made to indicate the use of standard engineering handbooks in connection with the calculations of strength of materials, and numerous references are made to tables and formulas that are to be found in works of that kind.

CHAPTER I

PRINCIPLES OF THE STRENGTH OF MATERIALS

It may be said, in a general way, that when designing machinery the designer must take into consideration two main factors. One is to so design the mechanism mechanically that the various motions required can be obtained by means of the machine. The other is to so proportion the parts that they will be strong enough to do the work for which they are intended without breakage and, in most cases, without distortion. A third factor also enters prominently into the design, that of so designing a machine that the various parts can be easily manufactured, but this last factor, while commercially fully as important as the other two, has not a direct bearing upon the actual working of the completed mechanism.

The subject that will be dealt with in this Reference Book relates to the second of the two main questions affecting the design, that of proportioning the parts so that they will be strong enough to properly do the work for which they are intended. This problem, in turn, may be divided into two sections, one of which deals with the determining of the forces which act upon a machine part, tending to break or distort it, and the determination of the actual proportions necessary to resist these forces. The science of determining the forces acting upon a machine part or, in general, upon an engineering structure, is termed *mechanics*, this word being used in its more limited sense, often referred to as *theoretical mechanics*. The problem of actually determining the dimensions of details of machines or structures with regard to their strength is covered by that part of mechanical knowledge which is known as the *strength of materials*. It should be understood, however, that the question of determining the forces acting upon an engineering structure or machine and the determination of the actual dimensions and materials required to resist these forces, are so closely connected that in dealing with the strength of materials we must also deal, to a very large extent, with the mechanical theory of forces.

When the forces acting upon a machine part are definitely known, it is, as a rule, comparatively easy to determine the actual proportions required to resist the action of these forces. There are, however, a number of instances in which the forces are applied in such a manner that it has so far proved impossible for the mathematician to determine with exact preciseness formulas that would cover each individual case, and many of the formulas used in calculating the strength of materials are based on results obtained by experiments and on practical experience. Of course, it must be understood at the outset that the actual strength of any material, such as steel, brass,

copper, etc., must have been experimentally determined before any calculations at all can be made that would give definite results. Hence, what is known as the *testing of the strength of materials* lies at the basis of all calculations of strength and endurance.

Important Definitions

There are a number of expressions used in connection with calculations of strength of materials that must be explained at the outset. A *stress* is a force acting within a material or machine part resisting deformation. A *load* is a force applied from without to a material. The load tends to produce deformation and is resisted by the stress which it creates within the body.

A *working load* is the maximum load applied to a material under ordinary working conditions. A *working stress* is the stress produced in the material by this working load. A *safe working stress* is the maximum permissible working stress under given conditions, as, for example, for a certain material.

The *ultimate strength* of a material is its breaking strength in pounds per square inch, in tension, compression or shearing, as the case may be.

The *total stress* is the sum of all the stresses caused at one section of the body, irrespective of its area in square inches; while the expression *stress*, *working stress*, or *intensity of stress* generally means the number of pounds stress per each square inch of section.

Analysis of the Forces that Act upon a Machine Member

As mentioned, it is necessary to analyze or determine the forces that act upon a machine member, in order to be able to determine the actual dimensions necessary to insure its strength to resist distortion or breaking. Ordinarily, only the actual load resting upon or transmitted through the machine element need to be considered, but, in many instances, as in the case of bridges, elevator ropes for deep shafts, beams, etc., the weight of the part itself must be taken into account. In other instances, frictional resistance and forces due to inertia caused by change of velocity, as well as centrifugal forces must be considered. This latter consideration is especially necessary in the case of flywheels or pulleys moving at high velocities. In some instances, stresses are caused by forces due to changes in the temperature, as when a metal part is constrained between other metal surfaces.

These loads may be applied in three different ways. They may be applied steadily in one direction, in which case we speak of a steady or dead load. They may be alternately applied and removed, the load being constantly in motion, in which case we speak of a live load. A live load may be applied first in one direction and then in the reverse direction, or it may be applied intermittently in one direction. It may be gradually applied and gradually relieved, or it may be suddenly applied, in which case we speak of the material as being subjected to shock or impact.

Ultimate Strength of Materials

The materials used in machine building are mainly metals, whereas, in civil engineering, wood, natural and artificial stones, as well as metals, are used. These materials may be subjected to a stress either in *tension*, that is when the forces acting upon the material are trying to pull it apart, or in *compression*, when the forces acting are trying to crush the material, or in *shear*, a stress which results either from a direct shearing action or a twisting or turning action. The strength of materials is usually measured in pounds per square inch. For example, when we say that structural steel has a strength of 60,000 pounds per square inch in tension, we mean that a bar of structural steel, the cross-section of which is one square inch, will, on an average, not break before subjected to a load trying to pull it apart, of 60,000 pounds.

Structural steel has also a strength of 60,000 pounds per square inch in compression, but many metals have not the same strength in tension as in compression. Cast iron, for example, has only a strength of 15,000 pounds per square inch in tension, whereas its strength in compression, on an average, is 80,000 pounds per square inch. These values are the ultimate strength of the metals. When used in structures or machine members, the metals must never be subjected to so severe a strain, but the actual load must be much less per square inch in order to provide for a factor of safety.

In calculations of the strength of materials, one of the first considerations, after an analysis of the forces acting upon the material has been made and the problem thus been mechanically determined, is to assume the average ultimate strength of the material used in the construction in pounds per square inch. As already mentioned, this assumption is based upon the experiments made on these materials by many investigators in the past, and a table is given herewith which shows the strength of the metals most commonly used. A table is also given showing the average ultimate strength of common materials other than metals. In all calculations the ultimate strength of the materials may be taken from these tables, but in order to make sure that there is a margin for safety, a suitable factor of safety must, of course, be assumed.

The Factor of Safety

If the ultimate strength of a material like machine steel is 60,000 pounds per square inch in tension or compression, and we subject it to a load of 10,000 pounds per square inch, a factor of safety of 6 is used; that is, the ultimate strength of the material is six times as great as the load to which the material is subjected. The factor of safety must be greater for moving loads than for dead or steady loads. It must also be greater if the load is applied suddenly and suddenly removed than if the load is constant at all times. In other words, when the load varies from zero to maximum in one direction, as shown in the accompanying table, "Factors of Safety," that is, when the load varies from no-load to full-load, the factor of safety must in-

ULTIMATE STRENGTH OF COMMON METALS: POUNDS PER SQUARE INCH

Material	Tension	Compression	Shear	Modulus of Elasticity
Aluminum	15,000	12,000	12,000	11,000,000
Brass, cast	24,000	30,000	36,000	9,000,000
Bronze, gun-metal	32,000	20,000	——	10,000,000
Bronze, manganese	60,000	120,000	——	——
Bronze, phosphor	50,000	——	——	14,000,000
Copper, cast	24,000	40,000	30,000	10,000,000
Copper Wire, annealed	36,000	——	——	15,000,000
Copper Wire, unannealed	60,000	——	——	18,000,000
Iron, cast	15,000	80,000	18,000	12,000,000
Iron Wire, annealed	60,000	——	——	15,000,000
Iron Wire, unannealed	80,000	——	——	25,000,000
Iron, wrought	48,000	46,000	40,000	27,000,000
Lead, cast	2,000	——	——	1,000,000
Steel Castings	70,000	70,000	60,000	30,000,000
Steel, structural	60,000	60,000	50,000	29,000,000
Steel Wire, annealed	80,000	——	——	29,000,000
Steel Wire, unannealed	120,000	——	——	30,000,000
Steel Wire, crucible	180,000	——	——	30,000,000
Steel Wire, plow	268,000	——	——	——
Steel Wire, susp. bridge	200,000	——	——	30,000,000
Steel Wire, piano	300,000	——	——	——
Tin, cast	3,500	6,000	——	4,000,000
Zinc, cast	5,000	20,000	——	13,000,000

AVERAGE STRENGTH OF COMMON MATERIALS OTHER THAN METALS

Material	Compression	Tension
Bricks, best hard	12,000	400
Bricks, light red	1,000	40
Brickwork, common	1,000	50
Brickwork, best	2,000	300
Cement, Portland, one month old	2,000	400
Cement, Portland, one year old	3,000	500
Concrete, Portland	1,000	200
Concrete, Portland, one year old	2,000	400
Hemlock	4,000	6,000
Pine, short leaf yellow	6,000	9,000
Pine, Georgia	8,000	12,000
Pine, White	5,500	7,000
White Oak	7,000	10,000

FACTORS OF SAFETY

Material	Steady Load	Load Varying from Zero to Maximum in one Direction	Load Varying from Zero to Maximum in both Directions	Suddenly Varying Loads and Shocks
Cast Iron	6	10	15	20
Wrought Iron	4	6	8	12
Steel	5	6	8	12
Wood	8	10	15	20
Brick	15	20	25	30
Stone	15	20	25	30

crease as compared with that for a steady load. Now, if the load varies from a maximum in one direction to no-load, and then to a maximum in the other direction, as, for example, in a rod which is first submitted to a pulling or tensional stress, then to no load at all, and then to a crushing or compression stress, the factor of safety must again be increased. If the loads vary very suddenly, taking the nature of shocks, a very high factor of safety is required, even with the most reliable materials. This will be understood by studying the figures in the table "Factors of Safety."* The factor of safety required with various materials also differs. It must, for example, be greater for cast iron than for wrought iron because cast iron is not so dependable a material.

The following factors of safety are given as a general guide, and may be used under ordinary conditions:

Boilers, from 4½ to 6; piston- and connecting-rods for double-acting engines, from 12 to 18; piston- and connecting-rods for single-acting engines, from 9 to 12; shafts carrying flywheels, armature, etc., from 7 to 9; mill shafting, 24; steel work in buildings, 4; steel work in bridges, 5; steel work, generally, 6; cast-iron wheel-rims, 20; steel wheel-rims, 8.

Influence of Temperature on the Strength of Metals

The degree of temperature to which a machine or a structural member made from metal is subjected has a considerable influence upon its strength. If we assume that metals have what we might call a "normal strength" at 70 degrees F., we will find upon investigation that this strength often increases with an increase in temperature up to a certain degree, and then rapidly decreases with further

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* In an article published in MACHINERY in January, 1906, Prof. F. E. Cardullo analyzes the nature of the factor of safety in a systematic manner, and the method given below is abstracted from and based upon this article.

The factor of safety may be considered as the product of four primary factors which may be designated as factors a, b, c and d. Designating the factor of safety by F,

$$F = a \times b \times c \times d.$$

The first of these factors, a, is the ratio of the ultimate strength of the material to the elastic limit, meaning, in this case, by the elastic limit, that boundary line within which the material is perfectly elastic and takes no permanent set. For ordinary materials, the factor a is 2; for nickel steel and oil tempered forgings, it is reduced to 1½.

The second factor b depends on the character of the stress within the material. This factor is 1, for a dead load; 2, for a load varying between zero and maximum; and 3, for a load which produces alternately a tension and a compression equal in amount.

The third factor c depends upon the manner in which the load is applied to the piece under stress. For a load gradually applied, this factor is 1. For a load suddenly applied, the factor is 2. If the load is applied not only suddenly but with impact, this factor must be still further increased in value.

The last factor d may be called the factor of ignorance. The other factors provide against known conditions and this provides against the unknown. It commonly varies in value between 1½ and 3 and occasionally should be given as high a value as 10. It provides against accidental overload, against unexpectedly severe service and unreliable or imperfect materials, etc. When all the conditions are thoroughly known and there is no danger of overload, this factor may be made equal to 1½ for wrought iron and mild steel, and 2, for cast iron.

As an example of the use of the formula given for the factor of safety, find the factor of safety that ought to be used for a forged steel steam-engine piston-rod. The elastic limit will probably be slightly more than one-half the ultimate strength; hence, $a = 2$. The rod will be alternately in tension and compression; hence, $b = 3$. The steam pressure will be applied suddenly or nearly so; hence, $c = 2$. The material is of a reliable kind; hence, $d = 1\frac{1}{2}$. Then: $F = 2 \times 3 \times 2 \times 1\frac{1}{2} = 18$.

increase in temperature. For example, if the strength of wrought iron is assumed to be 100 per cent at 70 degrees F., it will be found to rise to 112 at 400 degrees F., and to 116 at 570 degrees F., but from this point it falls to 96 per cent at 750 degrees F., to 76 per cent at 930 degrees F., to 42 per cent at 1100 degrees F., until at 1475 degrees F. only 15 per cent of the original strength remains. Structural steel shows a maximum strength of 132 per cent at 400 degrees F., from which point it falls to 86 per cent at 750, and 28 per cent at 1100 degrees F. The strength of copper falls immediately so that at 210 degrees it is only 95 per cent, at 570 degrees, 73 per cent, and at 930 degrees, 42 per cent of the original strength. Bronze falls even more rapidly, retaining at 570 degrees only 57 per cent of its strength at 70 degrees F. Cast iron seems to retain its strength up to about 550 degrees F., from which point it falls off to 75 per cent at 930 degrees F. and 42 per cent at 1100 degrees F. Cast steel reaches its maximum strength at about 400 degrees F. and has only 57 per cent of its strength left at 930 degrees F.

Elasticity and Elastic Limit

When external forces act upon a material they produce stresses within it as mentioned. These stresses are fundamentally tension, compression or shearing stresses, although we sometimes speak of bending or torsional stresses. Bending stresses, however, are only a combination of tension and compression stresses, and possibly also of shearing stresses, as will be explained later. Torsional stresses are merely shearing stresses. In most instances, a combination of two or more of these stresses is produced, especially in machine parts. In structural designs, such as bridges, for example, it is quite common that members are subjected to tension only or compression only, but in machine parts simple stresses of this kind are not as frequently met with, especially if the parts enter into the moving mechanism. All stresses to which a material is subjected tend to cause a deformation in it. If the stress is not too great, however, the material will return to its original shape and dimensions when the external load is removed. The property which enables a material to return to its original shape and dimensions is called *elasticity* and differs greatly in different materials. Of the metals, lead, for example, has little or no elasticity, whereas the elasticity of steel is great by comparison.

If a material has been subjected to such a load that upon its removal the material cannot fully return to its original shape and dimensions, it is said that it has been stressed beyond its *elastic limit*. Up to the elastic limit deformation is directly proportional to the load, but when the elastic limit has been reached, and the load is still increasing, the deformation will cease to be proportional to the stress, although the material will not actually break before a much greater load has been applied. The elastic limit is difficult to determine with accuracy, although in iron and steel which has not been heat-treated, it is frequently about one-half of the ultimate strength of the material. In all engineering designs, the loads applied to the material must

never be so great that the elastic limit is ever exceeded. If it is, there will be a permanent set in the material which naturally interferes both with the action of a machine and with its safe operation.

Modulus of Elasticity

The *modulus of elasticity* is another expression used to determine a certain quality in materials of engineering which is of great importance in the calculations of strength. The modulus of elasticity of a material may be defined as the quotient obtained by dividing the stress per square inch by the elongation in the length of one inch caused by this stress. The modulus of elasticity is quite generally denoted by E. Suppose that a steel bar 10 inches long is subjected to a load of 45,000 pounds per each square inch of cross-section of the bar, and assume further that in the total length of 10 inches the bar elongates under its load 0.015 inch; then the elongation for each inch of length would be 0.0015 inch and the modulus of elasticity would be obtained as follows:

$$E = \frac{45{,}000}{0.0015} = 30{,}000{,}000.$$

As the elongation e is assumed to be proportional to the load up to the elastic limit, the modulus of elasticity of a material E may be used for finding the elongation per each inch produced by any load per square inch S, according to the following formula:

$$e = \frac{S}{E}.$$

For example, the modulus of elasticity of wrought iron is 27,000,000. Find the elongation per inch produced by a stress of 15,000 pounds per square inch.

$$e = \frac{15{,}000}{27{,}000{,}000} = 0.00055 \text{ inch.}$$

Elongation and Reduction of Area

When a piece of material is tested for tensile strength in a testing machine, it elongates a certain amount before rupture takes place. This elongation constitutes an important quality in the material, as it indicates its toughness or the degree to which the material is likely to give warning before it will actually break. It is measured or recorded as the percentage or stretch or elongation occurring in a given length of the original piece; this length is frequently assumed as two inches. For example, if a test piece two inches long is found to be 2¼ inches long after rupture, the elongation in two inches is said to be 12½ per cent. It should be noted that the recorded value of elongation for any test depends largely upon the original length selected for comparison, because the total elongation consists partly of a general extension which takes place mainly before the ultimate stress has been reached and which is distributed fairly uniformly over the whole length of the piece, and partly of an elongation in the vicinity of the section where the

rupture will occur, where the local elongation is much greater, and practically independent of the total length of the piece. At this point, the elongation is also accompanied by a marked contraction of cross-sectional area. The elongation at the time of rupture cannot be calculated, but, in every case, is found by actual tests.

As mentioned, a piece of material tested to failure in tension contracts or decreases in cross-sectional area at the point of rupture. The percentage of decrease of area in relation to the original normal cross-section is known as "reduction of area." For example, if the original cross-sectional area of a bar was 0.78 square inches and the section, after having tested the piece to failure, was 0.44 square inches, then the decrease of area would be 0.34 square inches and the reduction of area would be $0.34 \div 0.78 = 0.44$, or 44 per cent. The area of a round bar tested to destruction is usually computed from the mean of two diameters measured at right angles to each other.

Brittle materials fail without appreciable deformation. Thus the percentage of elongation and the reduction of area in test pieces of brittle materials are very small. As an example may be mentioned cast iron, which will break with practically no deformation.

Strength of Alloy Steels

Special alloy steels can, by proper heat-treatment, be made to show an increase of strength several hundred per cent greater than that of ordinary machine steel. It should be noted, however, that the great gain in strength of alloy steels is obtained only by certain heat-treatments. In the annealed condition, special alloy steels are but little superior to an ordinary carbon steel. As an example of what can be obtained by heat-treatment may be mentioned that while a carbon steel containing about 0.50 per cent carbon can by heat-treatment be made to attain an elastic limit up to 100,000 pounds per square inch, under the very best conditions, a nickel steel with about 3.5 per cent nickel may be heat-treated to attain an elastic limit of 200,000 pounds per square inch, under the most favorable conditions. Nickel-chromium steels have been heat-treated to show an elastic limit of from 175,000 to 250,000 pounds per square inch. Chrome-vanadium steels with proper heat-treatment may attain an elastic limit up to 225,000 pounds per square inch. A curious result of heat-treatment is that the elastic limit can be brought up much closer to the ultimate strength than in the case of unannealed or untreated materials. As an example may be mentioned a chrome-vanadium steel which in its annealed state has an elastic limit of 94,000 pounds per square inch and an ultimate tensile strength of 134,000 pounds per square inch. After having been heat-treated this steel possesses an elastic limit of 191,500 pounds per square inch and an ultimate strength of 210,000 pounds per square inch. In general, it is uncertain to base calculations relating to the strength of alloy steels on anything except actual tests performed with specimens having the exact composition and which have been given the exact heat-treatment that the parts calculated for will be given.

CHAPTER II

TENSION, COMPRESSION AND SHEARING STRESSES

The simple stresses, to which all other stresses in materials may be reduced, are tension, compression and shearing stresses. Bending and torsional stresses, as well as so-called "compound" stresses, are merely special forms or combinations of these three simple types of stresses.

With the various expressions used in the calculations of the strength of materials now defined, it is possible to take up the specific cases of calculations, considering one at a time. The simplest case is that of direct tension.

Tensional Stresses in Materials

Assume that a load is applied at the end of a bar, as indicated in the accompanying engraving Fig. 1. The stress in the bar caused by the load is assumed to be evenly distributed over the whole cross-sectional area of the bar. Hence, the stress per square inch in the section will equal the total load divided by the number of square inches in the bar, or:

$$\text{Stress per square inch} = \frac{\text{Load}}{\text{Area}}.$$

If S = permissible working stress in pounds per square inch;
P = total load in pounds;
A = area of cross-section in square inches.

Then
$$S = \frac{P}{A}.$$

Example.—A wrought-iron bar supports in tension a load of 40,000 pounds. (See Fig. 2.) The load is gradually applied, and then after having reached its maximum value, is gradually removed. Find the diameter of round bar required.

From the table "Ultimate Strength of Common Metals," the ultimate strength of wrought iron is found to be 48,000 pounds per square inch in tension. From the table "Factors of Safety," we find that a load varying from zero to maximum in one direction, gradually applied, requires a factor of safety of 6 for wrought iron. Hence, the safe working stress, in this case, would be $48{,}000 \div 6 = 8000$ pounds per square inch. Inserting known values in the given formula, we find:

$$S = \frac{P}{A};\quad 8000 = \frac{40{,}000}{A};\quad A = \frac{40{,}000}{8000} = 5.$$

The cross-section of the bar, hence, must be about 5 square inches. As the bar is circular in section, the diameter must then be about 2½ inches, the area of a circle of this diameter being 4.9 square inches.

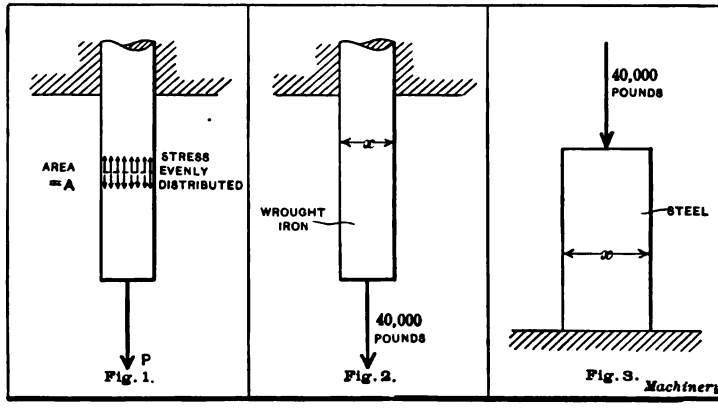
The elongation of a bar subjected to a load in tension is expressed by the formula:

$$e = \frac{P \times l}{A \times E}$$

in which e = elongation in inches;
P = total load in pounds;
l = length of bar in inches;
A = area of cross-section of bar in square inches;
E = modulus of elasticity.

Example.—What would be the total elongation of a bar supporting in tension a load of 40,000 pounds, assuming that the cross-sectional area of the bar is 5 square inches and that the bar is 5 feet long?

If we insert the known values in the formula just given, we will find the elongation. In this case, $P = 40{,}000$; $l = 5 \times 12 = 60$; $A = 5$;



Figs. 1 to 3. Examples of Tension and Compression

and E (obtained from the table "Ultimate Strength of Common Metals") for wrought iron-= 27,000,000.

Then:

$$e = \frac{40{,}000 \times 60}{5 \times 27{,}000{,}000} = 0.018 \text{ inch.}$$

Compression

If a material is subjected to compression, the load is assumed as being equally distributed over the total cross-sectional area and the formula for compression has exactly the same form as the formula for tension. If S, P and A have the same meaning as for tensional stress:

$$S = \frac{P}{A}.$$

Example.—A short structural steel bar supports in compression a load of 40,000 pounds. See Fig. 3. The load is steady. Find the diameter of the bar required.

From the tables, we find that structural steel has an ultimate strength of 60,000 pounds per square inch in compression and that steel subjected to a steady load should have a factor of safety of 5. Hence, the safe working stress equals $60{,}000 \div 5 = 12{,}000$ pounds per square inch. Inserting the known values in the formula:

$$12{,}000 = \frac{40{,}000}{A}, \text{ or } A = \frac{40{,}000}{12{,}000} = 3.33 \text{ square inches.}$$

The diameter of a bar, the cross-section of which is 3.33 square inches is 2 1/16 inches approximately.

The compression or shortening of a material subjected to a compressive stress corresponds to the elongation of a bar subjected to a tensile stress, and follows the same laws. Hence, let

e = shortening in inches of bar subjected to compression;
P = total load in pounds;
l = total length of bar in inches;
A = area of cross-section of bar in square inches;
E = modulus of elasticity.

Then,

$$e = \frac{P \times l}{A \times E}.$$

Example.—If the bar in the example just given were 8 inches long, how much would it be compressed or shortened by a load of 40,000 pounds, under the conditions stated?

In this case, $P = 40{,}000$; $l = 8$; $A = 3.33$; and E, from table, $= 29{,}000{,}000$. Hence,

$$e = \frac{40{,}000 \times 8}{3.33 \times 29{,}000{,}000} = 0.003 \text{ inch, approximately.}$$

The formulas that apply to compression should be used only if the length of the member being compressed is not greater than 6 times the least cross-sectional dimension. For example, these formulas should be applied to round bars only when the length of the bar is less than 6 times the diameter. If the bar is rectangular in shape, they should be applied only to bars that have a length less than 6 times the shortest side of the rectangle. When bars are longer than this, a compressive stress causes a sidewise bending action, and an even distribution of the compression stresses over the total area of the cross-section is no longer to be depended upon. Special formulas for long bars or columns will be given later. As a rule, bars subjected to simple tension or compression should be made round or square to insure an even distribution of the stress. If they are made of irregular section, particularly if they are made thin in one direction and wide in the other, the load will not be evenly distributed over the whole section, but may be concentrated at one edge, in which case this edge may be over-stressed while the other edge takes but little of the load.

While in the case of members subjected to compression the length of the member is limited, members subjected to tension can have any

length, providing the weight of the member itself is not so great as to enter into the problem. Whether the member is short or long, the stress will be evenly distributed in tension, and the safe stress will not be exceeded; but in instances where the elongation is a factor to be taken into account it is evident that the total elongation of a long bar subjected to tension will be greater than that of a short bar.

Shearing Stresses

The pin *E* shown in the illustration, Fig. 4, is subjected to shear. Parts *G* and *B* are held together by the pin and tend to shear it off at *C* and *D*. The areas resisting the shearing action are equal to the

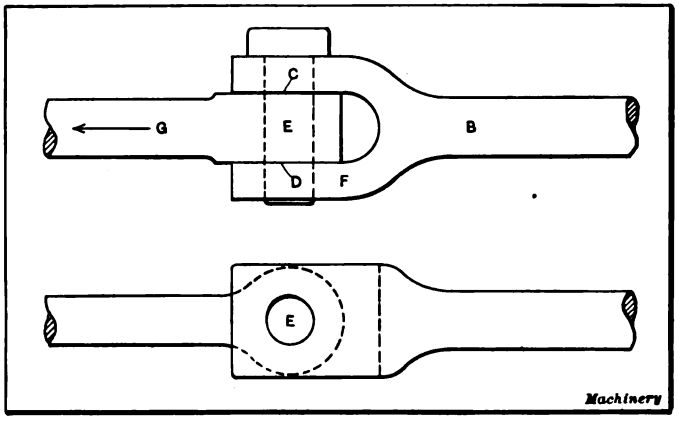


Fig. 4. Connecting-rod with Pin subjected to Shearing Stresses

cross-sectional areas of the pin at these points. The general formula for shear is

load = cross-sectional area × working stress.

Expressed as a formula, if

P = total load in pounds;

A = area of cross-section in square inches;

S = permissible working stress in pounds per square inch;

then,

$$P = A \times S.$$

The permissible working stress for shear is assumed as four-fifths or five-sixths of the permissible working stress in tension.

If a pin is subjected to shear as in Fig. 4, so that two surfaces, as at *C* and *D*, must fail by shearing before breakage takes place, the areas of both surfaces must be taken into consideration when calculating the strength. The pin is then said to be in *double shear*. If the lower part *F* of connecting-rod *B* were removed, so that member *G* were connected with *B* by a pin subjected to shear at *C* only, the pin would be said to be in *single shear*.

Example.—Assume that in Fig. 4 the load at *G* pulling on the connecting-rod is 20,000 pounds. The material of the pin is steel. The load varies from a maximum in one direction to a maximum in the

other direction, and is applied in such a manner that shocks are liable to occur. Find the required dimensions for the pin.

The load P is 20,000 pounds. The ultimate strength of steel in shear is 50,000 pounds per square inch. As shocks are liable to occur, the factor of safety to be used in this case is 12. (See the tables given for ultimate strength and factor of safety.) Hence the safe working stress $S = 50{,}000 \div 12 = 4170$ pounds per square inch. Inserting the known values in the formula given, we have:

$$20{,}000 = A \times 4170, \text{ or } A = \frac{20{,}000}{4170} = 4.8 \text{ square inches.}$$

As the pin is in double shear, that is, as there are two surfaces C and D over which the shearing stress is distributed, each of them must have an area of one-half the total shearing area A. In this case, then, the cross-sectional area of the pin will be 2.4 inches, and the diameter of the pin, to give a cross-sectional area of 2.4 inches, must be 1¾ inch.

Stresses in Machine Parts due to Temperature

If a bar of metal is confined in a space so that it is prevented from expanding or contracting, stresses will be induced in it if it is subjected to temperature changes. These stresses are termed *temperature stresses*, and their magnitude is measured by the amount of compression or elongation. In general, it is necessary to avoid temperature stresses in all designs of machinery and structures, and, if possible, allowance should be made for the expansion and contraction of members due to temperature changes. The stress per square inch produced by temperature changes may be determined by the following formula:

$$S = C \times T \times E$$

in which S = stress produced in the member by temperature change, in pounds per square inch;

C = coefficient of linear expansion of the metal under consideration;

T = the change in temperature in degrees F.;

E = modulus of elasticity.

The value of the coefficient of linear expansion for 1 degree F. equals 0.0000074 for high-carbon steel; 0.0000065 for machine steel; 0.0000062 for cast iron; and 0.0000068 for wrought iron.

Example.—What is the stress induced in a machine-steel machine member confined in a space so that it is prevented from expanding, if its temperature is raised from room temperature of 68 degrees F. to 150 degrees F.?

The modulus of elasticity E for machine steel may be assumed at 29,000,000. The change in temperature in degrees F. equals

$$150 - 68 = 82 = T.$$

C, the coefficient of linear expansion, as given above, is 0.0000065; hence, by inserting the known values in the formula given, we have:

$S = 0.0000065 \times 82 \times 29{,}000{,}000, = 15{,}500$ pounds per square inch, approx.

It will thus be seen that a stress of considerable magnitude can be induced in a metal member simply by a temperature change, if the member is not free to expand under the influence of the increased temperature.

Shape of Machine Parts

While the size of a machine part depends mainly upon the magnitude of the stresses to which it is subjected, its shape depends, to a large extent, upon the manner or direction in which the load or strain is brought to bear upon it. If a part is subjected to tension only, that is, if it merely resists a force tending to pull it apart, the shape is not very material, but, in general, a round bar is the best. This shape is also most compact and generally the cheapest. Almost any other shape is satisfactory, however, although it is well to avoid using parts which are thin in one direction and wide in the other, because, as already mentioned, the strain may then be brought to bear upon one edge instead of being uniformly distributed over the whole area, and a stress may be caused in one part of the cross-section which will be greater than that for which the material is adapted. The material may then begin to rupture at one edge and may thus gradually break, although the cross-sectional area, as a whole, would be large enough, if properly shaped, to carry the load imposed upon it.

When a machine part is designed to resist compression, it should, in general, have a shape similar to that required for resisting tension. There are, however, exceptions to this statement as, for instance, when the proportion of the length of the bar to its diameter or thickness is such that the member is likely to buckle or bend. As mentioned, this will take place in many cases when the length exceeds six times the diameter or the smallest distance across the member, and, in such cases, it may become desirable to use hollow or cross-ribbed forms of construction so as to bring the metal that resists compression as far from the axis of the member as possible; thus a hollow cylindrical form is very effective to resist compression, and a hollow square or cross-ribbed form may also be adopted for reasons of appearance or cheapness of production.

Stresses in Castings

Stresses in castings, due to shrinkage in cooling, often increase, to a considerable extent, the stresses due to the load. If all parts of a casting could be made to cool equally fast there would be little trouble from this source, but as different parts of the casting vary in thickness, the time required for cooling varies and stresses are set up which are sometimes great enough to rupture the casting without any additional load being placed upon it. In the case of a pulley, the hub, in cooling, tends to draw the arms away from the rim. As these strains are primarily due to uneven cooling, it is evident that in order to reduce them to the lowest point it is necessary to make the different parts of the casting of as nearly uniform thickness as possible. This is not always feasible, and in cases where it is not, a

liberal allowance should be made for the internal stresses by using a larger factor of safety when calculating for the external load.

Shrinkage strains often become so serious that it is necessary to make castings in two or more parts, which it would otherwise be possible to make at much less expense in one piece. As examples of this may be mentioned large jacketed cylinders for steam and gas engines. If these are cast in one piece the shrinkage stresses, together with the stresses set up by the varying temperatures incident to the service of these parts, are often sufficient to crack them.

Sometimes, however, as mentioned on page 33 of MACHINERY'S Reference Book No. 14, it is possible to utilize the shrinkage stresses to advantage. For instance, when cast iron was the standard material for the manufacture of ordnance, guns were cast with cores through which water was circulated so as to cool the surface of the bore before the outer part solidified. When the gun was fired the inner layers of metal stretched more than the outer ones. By cooling the inner layers of the metal first, shrinkage strains were induced in the walls of the guns, causing the outer layers of the metal to compress the inner ones. The combined effect of the shrinkage stresses and the stresses produced by the explosion was to produce a uniform stress throughout the walls of the guns and thus to reduce the chance of rupture. A similar condition is taken advantage of in the manufacture of modern ordnance, except that in this case steel is used as the material and various rings or layers are shrunk the one upon the other. The stresses induced by shrinking one layer upon the other are counteracted at the moment of firing by the stress induced by the explosion of the charge.

It is not often, however, that advantage can be taken of shrinkage strains in this way. More often are they troublesome and cause the work to warp in the process of machining as well as causing unexpected breakage at what seems insignificant loads. The designer must take as much care in proportioning the cast-iron parts so that as small shrinkage stresses as possible are induced as he does in calculating the correct proportions to resist the stresses caused by the external load. In order that unequal contraction shall not produce dangerous stresses in the metal, it is necessary, in addition to making the parts as nearly uniform in thickness as possible, to avoid sharp corners and to see that the various parts shall be free to expand when necessary.

CHAPTER III

BEAMS AND BENDING STRESSES

When a body is subjected to bending, it is supported at one or more points and a load is applied at a point that is not directly supported, thus causing a bending action. Parts of machines and structures subjected to bending are known mechanically as *beams*. Hence, in this sense, a lever fixed at one end and subjected to a force at its other end, a rod supported at both ends and subjected to a load at its center, or the overhanging arm of a jib crane, would all be known as beams. The theory of the stresses in beams and the method of determining them is a subject that is not as simple as when materials are subjected to tension or compression only. It will be necessary to go into considerable details to explain the action of the forces upon beams and the method of determining the dimensions required for resisting these forces.

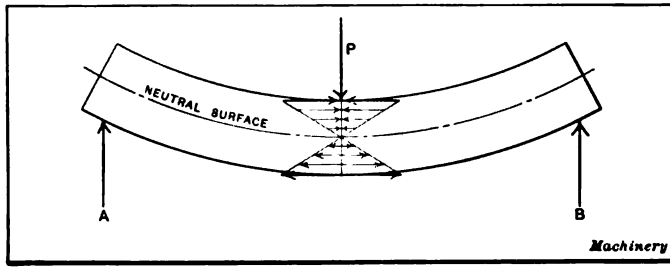


Fig. 5. Exaggerated Example, showing Effect of Bending Stresses in Beams

The stresses in a beam are principally tension and compression stresses. In Fig. 5 is shown a beam supported at *A* and *B*, a load is applied at *P*, and the beam is shown bent or deflected in an exaggerated manner in order to illustrate the principles involved. The lower fibers are stretched by the bending action and are thus subjected to a tensile stress, while the upper fibers are compressed and thus subjected to a compressive stress. There will be a slight lengthening of the fibers in the lower part of the beam, while those on the upper side will be somewhat shorter, depending upon the amount of deflection. If we assume the beam illustrated to be of round or square cross-section, there will be a layer or surface through its center line which will be neither in compression or in tension. This surface is known as the neutral surface. In metals and similar materials this neutral surface passes through the center of gravity of the cross-section, as long as the material is not stressed beyond the elastic limit. It is evident that this neutral surface divides the beam

into two sections, one of which is located above and one below it, those sections above it in the illustration being all in compression, while those beneath are all in tension. The stresses of the individual layers or fibers of the beam will be proportional to their distances from the neutral surface, the stresses being greater the further away from the neutral surface the fiber is located. Hence, there is no stress on the fibers in the neutral surface, but there is a maximum tension on the fibers at the extreme lower side and a maximum compression on the fibers at the extreme upper side of the beam. In calculating the strength of beams, it is, therefore, only necessary to determine that the fibers of the beam which are at the greatest distance from the neutral surface are not stressed beyond the safe working stress of the material. If this is the case, all the other parts of the section of the beam are not stressed beyond the safe working stress of the material.

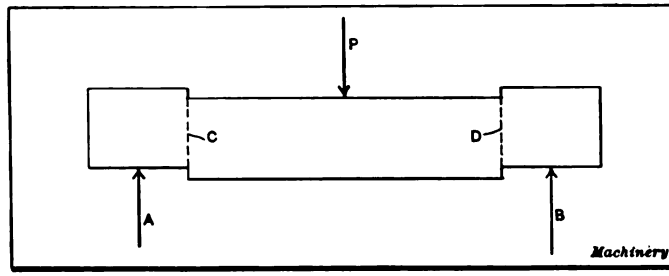


Fig. 6. Diagrammatical Illustration showing Action of Shearing Stresses in a Beam subjected to Load

Shearing Stresses in Beams

In addition to the tension and compression stresses mentioned in the previous paragraphs, a loaded beam is also subjected to a stress which tends to shear it, as indicated at *C* and *D* in Fig. 6. This shearing stress equals the load to which the beam is subjected. In most cases, the shearing action can be ignored for metal beams, especially if the beams are long and the loads far from the supports. If the beams are very short and the load quite close to a support, then the shearing stress may become equal to or greater than the tension or compression stresses in the beam and in that case the beam should be calculated for shear in the manner already explained under "Shearing Stresses."

Reaction at the Supports

When a beam is loaded by vertical loads or forces, the sum of the reactions at the supports equals the sum of the loads. In a simple beam, when the loads are symmetrically placed with reference to the supports, or when the load is uniformly distributed, the reaction at each end will equal one-half of the sum of the loads. When the loads are not symmetrically placed, the reaction at each support may be ascertained from the fact that the algebraic sum of the moments must

equal zero. In the accompanying illustration, Fig. 7, if moments are taken about the support to the left, then:

$$R_2 \times 40 - 8000 \times 10 - 10{,}000 \times 16 - 20{,}000 \times 20 = 0;$$

$$R_2 = 16{,}000 \text{ pounds.}$$

Moments taken about the support at the right will, in the same way, give

$$R_1 = 22{,}000 \text{ pounds.}$$

The sum of the reactions equals 38,000 pounds, which is also the sum of the loads. If part of the load is uniformly distributed over the beam, this part is first equally divided between the two supports, or the uniform load may be considered as concentrated at its center of gravity.

Example.—In Fig. 7 there is a uniformly distributed load of 30,000 pounds on the beam between the supports, in addition to the loads 8000, 10,000, and 20,000 pounds indicated in the engraving. Find the reactions at the support.

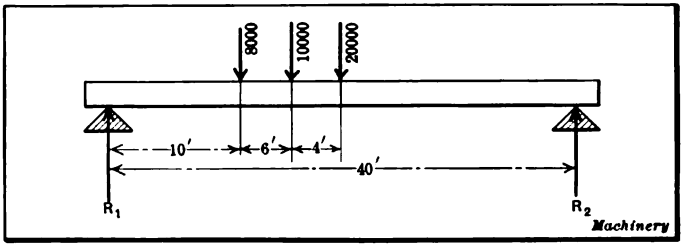


Fig. 7. Diagrammatical View Illustrating the Principle of Reaction at the Supports

The uniformly distributed load adds 15,000 pounds to the reaction at each support. We have already found $R_1 = 22{,}000$ pounds and $R_2 = 16{,}000$ pounds, in case the uniform load is not considered. Hence, with uniform load:

$R_1 = 22{,}000 + 15{,}000 = 37{,}000$ pounds.

$R_2 = 16{,}000 + 15{,}000 = 31{,}000$ pounds.

Bending Moments

The bending action of a load upon a beam is called the *bending moment.* For example, in Fig. 8 the load P acting downward on the free end of the cantilever beam has a moment or bending action about the support at A equal to the load multiplied by its distance from the support. The bending moment is commonly expressed in inch-pounds, the load being expressed in pounds and the lever arm or distance from the support in inches. The length of the lever arm should always be measured in a direction at right angles to the direction of the load. Thus, in Fig. 9, the bending moment is not $P \times a$, but is $P \times l$, because l is measured in a direction at right angles to the direction of the load P.

The ability of a beam to resist the bending action or the bending moment is called the *moment of resistance* of the beam. It is evident

that the bending moment must be equal to the moment of resistance. The moment of resistance, in turn, is equal to the stress in the fiber furthest away from the neutral plane multiplied by the section modulus. The *section modulus* is a factor which depends upon the shape and size of the cross-section of a beam, and is given for different cross-sections in all engineering handbooks. (See MACHINERY'S HANDBOOK, page 308.) The section modulus, in turn equals the *moment of inertia* of the cross-section, divided by the distance from the neutral surface to the most extreme fiber. The moment of inertia for various cross-sections will also be found tabulated in standard engineering handbooks. Let,

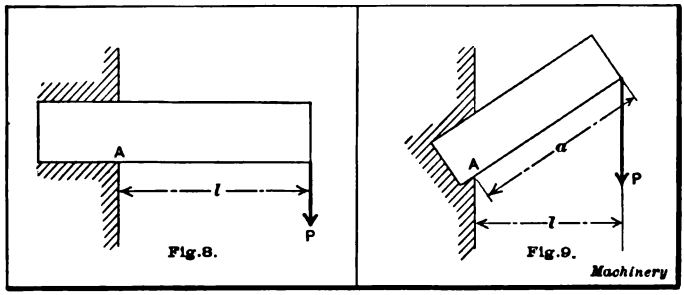
M_b = bending moment in inch-pounds;

S = permissible working stress in pounds per square inch;

Z = section modulus;

I = moment of inertia of cross-section;

y = distance from neutral plane to most remote fiber of cross-section.



Figs. 8 and 9. Views illustrating Principle of Bending Moments

Then the following formula may be given as the fundamental formula for bending of beams:

$$M_b = \frac{SI}{y} = SZ; \text{ and } S = \frac{M_b}{Z}.$$

The moment of inertia I is a property of the cross-section that determines its relative strength. In calculations of strength of materials, a standard engineering handbook is necessary because of the tabulated information of section moduli and moments of inertia, areas of cross-sections, etc., to be found therein.

The bending moments for beams supported and loaded in various ways are also given in standard handbooks. There are a great many different ways in which a beam can be supported and loaded, and the bending moment caused by a given load varies greatly according to whether the beam is supported at one end only or whether it is supported at both ends, and also according to whether it is freely supported at the ends or whether the beam is held firmly. Then the load may be equally distributed over the full length of the beam or may be applied at one point either in the center or near to one or the other of the supports. The point where the stress is maximum is gen-

erally called the critical point. In some handbooks, instead of giving the bending moment, the stress at the critical point, which equals bending moment divided by section modulus (see preceding formula) is given.

Examples of Strength of Beams

A number of examples of the strength of beams will illustrate the use of the formulas given and also of the formulas to be found in standard handbooks.

Example 1.—A square bar, held firmly at one end, is supporting a load of 3000 pounds at the outer free end. The length of the bar is 2½ feet. The bar is made of structural steel and the load is steady. Find the size of bar required for safe loading.

M_b = load × lever arm in inches = 3000 × 30 = 90,000;

S = safe stress = 60,000 ÷ 5 = 12,000;

From MACHINERY'S HANDBOOK, page 308:

I, for a square = $s^4 \div 12$, if s = side of square;

y, in the present case = $s \div 2$.

Then,

$$90{,}000 = \frac{12{,}000 \times \frac{s^4}{12}}{\frac{s}{2}} = \frac{12{,}000 \times s^3}{6} = 2000s^3;$$

$$s^3 = \frac{90{,}000}{2000} = 45; \text{ and } s = 3.56 \text{ inches.}$$

Example 2.—A rectangular steel bar two inches thick and firmly built into a wall, as shown in Fig. 10, is to support 3000 pounds at its outer end 36 inches from the wall. What would be the necessary depth h of the beam to support this weight?

The bending moment equals the load times the distance from the point of support, or, 3000 × 36 = 108,000 inch-pounds.

The permissible fiber stress in the steel bar in this case may be found by using a factor of safety of 5, giving 60,000 ÷ 5 = 12,000 pounds per square inch. Hence, $M_b = 108{,}000$, and $S = 12{,}000$.

Inserting these values in formula, $M_b = SZ$, we have:

$$108{,}000 = 12{,}000\ Z, \text{ from which}$$

$$Z = \frac{108{,}000}{12{,}000} = 9.$$

Now from a standard handbook (MACHINERY'S HANDBOOK, page 308), we find that the section modulus for a rectangle equals $\frac{bd^2}{6}$, in which b is the length of the shorter side and d of the longer side of the present rectangle. Hence, $Z = \frac{bd^2}{6}$.

But $Z = 9$ and $b = 2$.

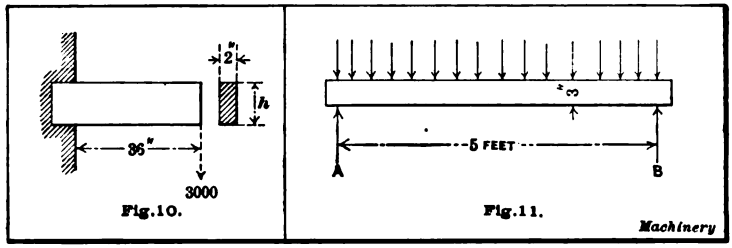
Inserting these values into the formula, we have:

$$9 = \frac{2d^3}{6},$$

from which $d^3 = 27$, and $d = 5.2$ inches. This value d corresponds to dimension h in Fig. 10. Hence, the required depth of the beam to support a load of 3000 pounds at the outer end with a factor of safety of 5 would be 5.2 inches.

Example 3.—A round steel bar is supported at A and B as shown in Fig. 11. It is uniformly loaded with a load of 20,000 pounds. The distance between supports is 5 feet. The steel bar is 3 inches in diameter. Assuming steel to have an ultimate tensile and compressive strength of 60,000 pounds per square inch, what factor of safety has been assumed in the case described?

From MACHINERY'S HANDBOOK, page 340, we find that for a bar sup-



Figs. 10 and 11. Examples of Problems met with in Beam Calculations

ported at the ends and uniformly loaded, the maximum stress (here termed S) is at the center and equals:

$$S = \frac{Wl}{8Z}$$

where W = total load in pounds, uniformly distributed;
l = length of beam in inches;
Z = section modulus.

In this case $W = 20{,}000$; $l = 5 \times 12 = 60$ inches; Z for bar 3 inches in diameter (see MACHINERY'S HANDBOOK, page 327) = 2.651. Hence,

$$S = \frac{20{,}000 \times 60}{8 \times 2.651} = 56{,}600 \text{ pounds per square inch.}$$

As the ultimate strength of the material is 60,000 pounds per square inch, this gives practically no factor of safety at all. The bar must either be relieved of part of the load, or a much larger bar must be used to support the given load. With a factor of safety of 5 the bar would not support much more than $20{,}000 \div 5 = 4000$ pounds in this case.

Strength of Channels

Experiments on standard channels carried out by Bach (published in 1909) show that the regular bending formula for beams freely supported at their ends and loaded in the center gives too high a value for the strength of structural channels. The experiments show

that the amount by which the value obtained from the formula is greater than that obtained by experiments, is, for channels 4¾ inches high, 7 per cent; for channels 8¾ inches high, 18 per cent; and for channels 11¾ inches high, 26 per cent. These values are those found when the load is assumed to be applied in the center line of the web of the channel as shown in Fig. 12. If the load is placed along the line of the vertical neutral axis of the channel as shown in Fig. 13, the permissible load according to the beam formula is 10, 25.5 and 34 per cent greater than that shown by the experiments. These experiments, therefore, indicate that when the usual formulas are employed in calculations, for channels or other structural shapes, a liberal factor of safety should be allowed in order to compensate for the difference of the results given by the formula and those of actual experiments. It should be noted that the formula for bending is fully correct whenever the section of the member is such that the load is fully distributed over the whole sectional area, as in a rectangular section,

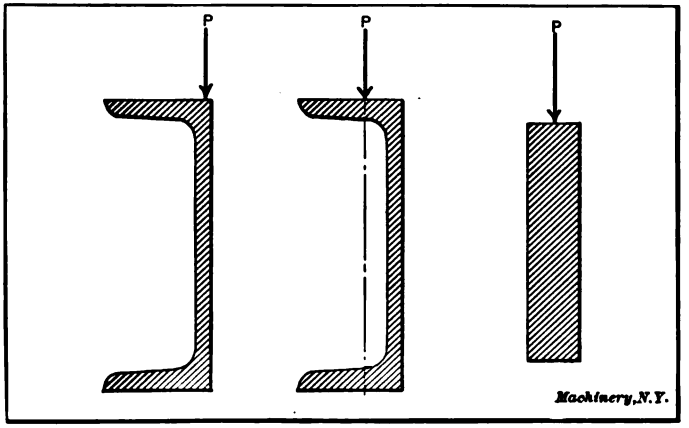


Fig. 12 Fig. 13 Fig. 14

Fig. 14; but in the case of channels as well as many other structural shapes, the load is not, as a rule, properly distributed over the whole section, but stresses certain portions of the section in a higher degree than others.

Deflection of Beams

When a beam is loaded, it will deflect a certain amount under a given load. The formulas for the deflection are generally quite complicated, but will be found for all ordinary conditions in the standard handbooks. For this reason, there is no need of entering upon the subject of deflection here, since all that could be done would be to repeat these formulas here; this would prove too voluminous for a treatise of the kind here attempted, the purpose of which is to give a general idea of the principles employed, and to act as a guide with relation to the use of the formulas and tables found in general engineering handbooks.

Bending Stresses Combined with Direct Tension or Compression

In U-shaped machine parts, such as, for example, punch or shear frames, the metal in the back of the frame resists both a uniformly distributed tensile stress due to the pressure between the jaws, and stresses due to the bending moment set up by the same pressure. In a punch or shear frame, the bending stresses will appear as tensile stresses towards the front of the frame and as compressive stresses at the back. The maximum stress in a case of this kind, therefore, is composed of the sum of the uniformly distributed tensile stress and the tensile stress due to the bending moment.

Should, again, the machine part be of such design and shape that the pressure exerted upon it is in such a direction that the stresses induced are partly compression and partly bending stresses, then the

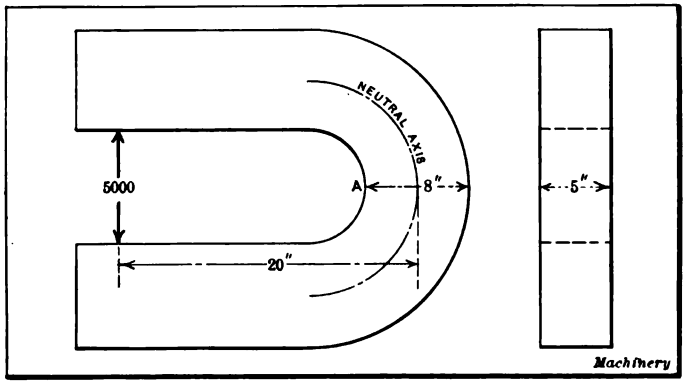


Fig. 15. Bending Stresses Combined with Direct Tension Stresses

stress due to direct compression should be added to the compressive stress due to bending in order to find the total compressive stress to which the machine part is subjected.

As a simple example illustrating this principle, assume that we have a U-clamp of rectangular cross-section, as illustrated in Fig. 15. A force of 5000 pounds tends to force the ends of the U-clamp apart, as indicated. What will be the tensile stress at *A*?

The dimensions of the cross-section of the clamp are given in the illustration.

The bending moment equals $5000 \times 20 = 100{,}000$ inch-pounds.

The section modulus equals $\frac{5 \times 8^2}{6} = \frac{5 \times 64}{6} = 53.3$.

Hence, from formula

$$S = \frac{M_b}{Z}$$

we have, by inserting the known values:

$$S = \frac{100{,}000}{53.3} = 1875 \text{ pounds per square inch.}$$

The direct tensile stress of 5000 pounds is uniformly distributed over the whole cross-sectional area; hence this stress equals

$$\frac{5000}{5 \times 8} = \frac{5000}{40} = 125 \text{ pounds per square inch.}$$

The total tensile stress at A is then:

$$1875 + 125 = 2000 \text{ pounds per square inch.}$$

Beams of Uniform Strength Throughout Their Length

As a rule, there is a certain point along a beam at which the bending moment is at its maximum, and at every other point along the beam the bending moment is less. Therefore, when a beam is of uniform cross-section, it is made strong enough to resist the bending moment at the section where the stress in the beam will be the greatest, and will as a result be excessively strong or have an excess of material at every other section. Sometimes it may be desirable to have the cross-section uniform on account of the simplicity of manufacture, as, for example, in the case of structural steel sections. In other cases, however, the metal can be more advantageously distributed if the beam is so designed that its cross-section varies from point to point, so that at every point it is just great enough to take care of the bending stresses at that point with an ample factor of safety. This construction is especially suitable for cast members and is also employed when members such as girders are built up from a number of different plates, I-beams and angles.

In the following a number of beams will be shown in which the load is applied in different ways and which are supported by different methods, and in which the shape of the beam required for uniform strength will be indicated. In the examples shown, it should be noted that the shape given is the theoretical shape required to resist bending only. It is apparent that sufficient cross-section of the beam must also be added either at the points of support, in the case of beams supported at both ends, or at the point of the application of the load, in the case of beams loaded at one end and supported at the other, to take care of the vertical shear.

In Fig. 16 is shown a beam held rigidly at one end and loaded at the other. The width of the beam is uniform. In order that the beam shall be of uniform strength throughout its length, its depth must decrease towards the loaded end. The outline of the beam shape will be that of a parabola with its vertex at the loaded end. If the outline is made like that of half a parabola, as shown in Fig. 17, the beam will also be of uniform strength throughout its length, and it may also be reversed so that the upper edge is parabolic, as shown in Fig. 18. In all these cases, let

P = carrying capacity in pounds;
S = safe working stress in pounds per square inch;
b = width of beam in inches;
h = height of beam in inches at support;
l = length of beam in inches.

Then the carrying capacity will be found from the following formula:

$$P = \frac{Sbh^2}{6l}.$$

A beam of approximately uniform strength, firmly secured at one end and loaded at the other, is shown in Fig. 19. The depth of the beam decreases towards the loaded end, the proportions of the beam being as indicated in the engraving. The same formula for the carrying capacity as given above for beams of parabolic shape is used for this beam also.

If the beam is loaded with a uniformly distributed load and the width of the beam is uniform, then the depth of the beam will decrease towards the outer end in such a manner that the outline of the beam will be triangular, as indicated in Fig. 20.

In this case, if the symbols in the formula denote the same quantities as in the formula previously given for beams of this class, the carrying capacity is found as follows:

$$P = \frac{Sbh^2}{3l}.$$

If the depth h of the beam is uniform, the load is uniformly distributed, and the beam held rigidly at one end, then the width of the beam will gradually decrease towards the point, as indicated in Fig. 21, the outline of the beam being formed by two parabolas which are tangent to each other at their apexes at the outer end of the beam. In this case also the formula for the carrying capacity is written as follows:

$$P = \frac{Sbh^2}{3l}.$$

Figs. 22 to 24 show beams supported at both ends. In Fig. 22 the load is concentrated in the center, the depth of the beam is uniform, the width of the beam is maximum at the point of loading, and from here the beam tapers towards both ends, as indicated.

In Fig. 23 the load is also concentrated at the center but the width of the beam is uniform and the variation for strength is made in the depth, the depth being maximum at the point of loading, and the outline of the beam being determined by two parabolas with apexes at the points of support.

In both of the above cases, the formula for the carrying capacity is:

$$P = \frac{2\,Sbh^2}{3l}$$

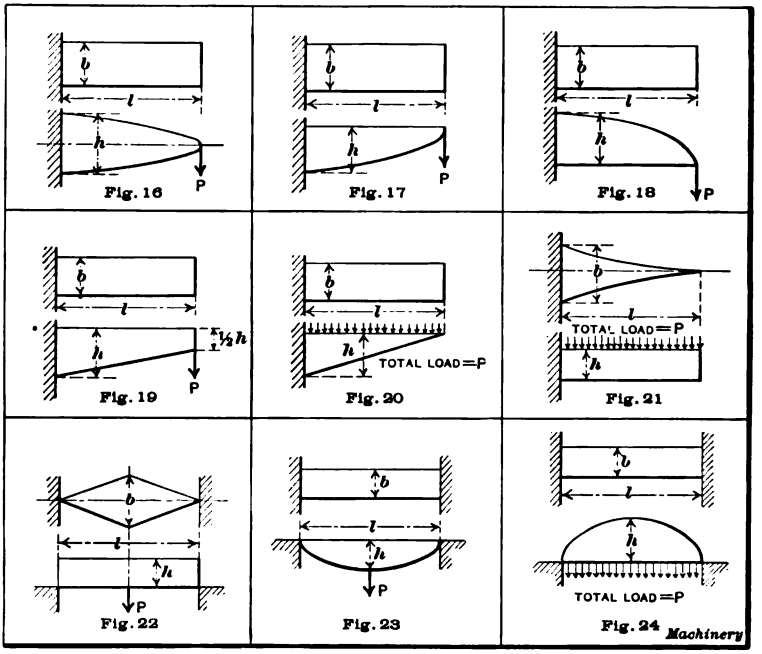
in which the symbols are the same as in the other formulas given for beams of uniform strength.

In Fig. 24 a beam is shown supported at both ends with the load uniformly distributed. The width of the beam is uniform and the depth of the beam is maximum at the center, the outline of the beam

in this case being one-half of an ellipse. The carrying capacity in this case is determined by the formula:

$$P = \frac{4\,S b h^2}{3l}.$$

A number of other cases of beams of uniform strength throughout their length will be found in more exhaustive works on beams and the strength of materials. A total of twelve different cases will be found in MACHINERY'S HANDBOOK, pages 352 and 353.

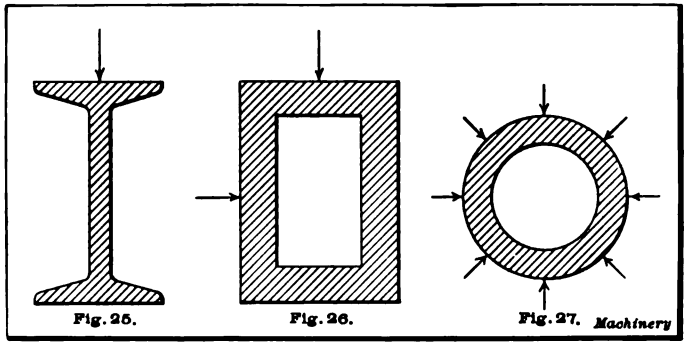


Figs. 16 to 24. Beams of Uniform Strength

It should be noted that the theoretical shapes of the beams shown in Figs. 16 to 24 are based on the stated assumptions of uniformity of width or depth of cross-section, and unless these are observed in the design, the theoretical outlines do not apply without modifications. For example, in a cantilever with the load at one end, the outline is a parabola only when the width of the beam is uniform. It is not correct to use a strictly parabolic shape when the thickness is not uniform, as, for instance, when the beam is made of an I- or T-section. In such cases, some modifications may be necessary; but it is evident that whatever the shape adopted, the correct depth of the section can be obtained by an investigation of the bending moment and the shearing load at a number of points, and then a line can be drawn through the points thus ascertained, which will provide for a beam of practically uniform strength whether the cross-section be of uniform width or not.

Cross-sectional Shape of Beams

When a member is designed to resist bending, it should always have its greatest depth of material in the direction in which the force is applied, because the capacity of a member to resist bending, as indicated by the formula for the section modulus, referred to on page 21, increases as the square of its thickness or depth in the direction of the force, and only directly as its width. For example, if the depth of a beam is increased two times in the direction of the force, its capacity to resist bending increases four times, whereas, if its width is increased two times, its capacity to resist bending is only increased two times. Again, if the depth in the direction of the force is increased three times, its capacity to resist bending is increased



Figs. 25 to 27. Cross-sectional Shape of Beams depending upon Method of Loading

nine times, while if it was increased three times in width only, the strength would increase only three times. The proportion of depth and width must, of course, not be carried to an extreme, as in that case there might be a tendency for the member to buckle or yield sideways.

When a material such as steel is used, which has practically the same properties in tension or compression, the most economical form of beam cross-section is an I-beam, if the beam is subjected to vertical loading only; that is, if a load in one direction only is applied to it, as indicated in Fig. 25. If, again, the beam is loaded in both a vertical and horizontal direction, that is, in two directions at right angles to each other, then a beam of hollow rectangular section, as shown in Fig. 26, will require the least amount of material for a given strength and stiffness. If the load is the same in the horizontal as in the vertical direction, then the cross-section would, of course, be a hollow square. For equal loading in any direction, a hollow circular section, as shown in Fig. 27, should be used. Pipe sections lend themselves exceedingly well for use in such instances.

CHAPTER IV

TORSIONAL STRESSES AND APPLICATIONS

Assume that a shaft A, as shown in Fig. 28, is subjected to a twisting action caused by the force P acting at the end of lever L. In that case the twisting moment is $P \times l$. This also is frequently called the torsional moment. The torsional moment must be equal to the moment of resistance in torsion which, in turn, equals the stress multiplied by the section modulus for torsion, similarly as in the case explained for bending. The section modulus for torsion, in turn, equals the polar moment of inertia divided by the distance from the center of gravity

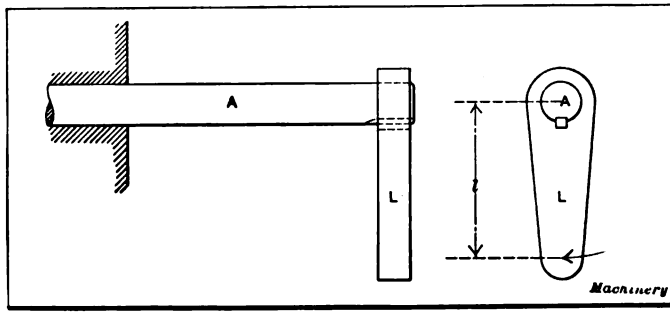


Fig. 28. Shaft Subjected to Torsional Stress

to the most extreme fiber, so that the fundamental formula for torsion may be given as:

$$M_t = SZ_p = S\frac{I_p}{y}; \text{ or } S = \frac{M_t}{Z_p}$$

in which M_t = moment of force tending to twist (torsional moment) in inch-pounds;

S = permissible working stress in pounds per square inch;

I_p = polar moment of inertia;

Z_p = section modulus for torsion;

y = distance from center of gravity to most remote fiber.

This formula holds true only for *circular* sections, but may be applied with fair accuracy also to sections which are *nearly circular*. For other sections the section modulus does not equal the polar moment of inertia divided by the distance from the center of gravity to the most remote fiber. In MACHINERY'S HANDBOOK, page 448, will be found a table of "Polar Moments of Inertia and Polar Section Moduli," for a number of cross-sections.

The polar section modulus or section modulus for torsion for a circle is that most commonly used and equals:

$$Z_p = \frac{3.1416 \times D^3}{16} = 0.196\, D^3$$

in which D = diameter of circle.

The permissible working stress for torsion is equal to the permissible working stress for shear, because the torsional stress is practically nothing but a shearing stress, and may, therefore, be assumed as about four-fifths or five-sixths of the permissible stress in tension.

Example.—A square bar is subjected to a steady torsional moment of 90,000 inch-pounds. The bar is made of structural steel. Find the size of bar required for safe loading.

In this case,

Torsional moment $M_t = 90{,}000$;

Safe working stress $S = 50{,}000 \div 5 = 10{,}000$;

Polar section modulus (see MACHINERY'S HANDBOOK, page 448) $= 0.22s^3$ for a square, if s is the side of the square.

Hence,

$$M_t = SZ_p, \text{ or } 90{,}000 = 10{,}000 \times 0.22s^3.$$

$$s^3 = \frac{90{,}000}{2200} = 40.9, \text{ and } s = 3.45 \text{ inches.}$$

The two most common applications of torsional strength met with in machine design are the strength of shafting and the carrying capacity of helical springs.

Application to Shafting

In the case of shafting, the formulas for torsional strength can be given in a simplified form, provided the shafting is always of circular cross-section. The twisting strength of a shaft may then be determined from the formulas:

$$T = PR = \frac{3.14\, d^3 S}{16} = \frac{d^3 S}{5.1}$$

or

$$d = \sqrt[3]{\frac{5.1\, PR}{S}} = \sqrt[3]{\frac{321{,}000 \text{ H.P.}}{nS}}.$$

in which T = twisting moment in inch-pounds;

P = force acting upon the shaft, producing rotation, in pounds;

R = length of lever arm of force P, in inches;

d = diameter of shaft in inches;

S = allowable torsional shearing stress in pounds per square inch;

n = number of revolutions of shaft per minute;

H.P. = horsepower to be transmitted.

Example.—Find the diameter of shaft required to transmit 60 horse-

power at 300 revolutions per minute, if the maximum safe stress of the material (special alloy steel) of which the shaft is made is 10,000 pounds per square inch.

The formula for finding the diameter of shaft is:

$$d = \sqrt[3]{\frac{321{,}000 \times \text{H. P.}}{nS}}$$

If we insert the given values, we have:

$$d = \sqrt[3]{\frac{321{,}000 \times 60}{300 \times 10{,}000}} = \sqrt[3]{6.42} = 1.86 \text{ inch.}$$

The shaft may, therefore, be made, say, 1⅞ inch in diameter.

The allowable stress for ordinary shafting may be assumed as 4000 pounds per square inch for main power-transmitting shafts; 6000 pounds per square inch for lineshafts carrying pulleys; and 8500 pounds per square inch for small, short shafts, countershafts, etc. The horsepower transmitted using these allowable stresses is as follows:

For main power-transmitting shafts:

$$\text{H.P.} = \frac{d^3n}{80}; \text{ or } d = \sqrt[3]{\frac{80 \text{ H.P.}}{n}}.$$

For lineshafts carrying pulleys:

$$\text{H.P.} = \frac{d^3n}{53.5}; \text{ or } d = \sqrt[3]{\frac{53.5 \text{ H.P.}}{n}}.$$

For small, short shafts:

$$\text{H.P.} = \frac{d^3n}{38}; \text{ or } d = \sqrt[3]{\frac{38 \text{ H.P.}}{n}}.$$

Shafting which is subjected to shocks, sudden starting and stopping, etc., should be given a greater factor of safety than is indicated by the allowable stresses just mentioned.

Example.—What would be the diameter of a lineshaft to transmit 10 horsepower? The shaft makes 150 revolutions per minute.

$$d = \sqrt[3]{\frac{53.5 \times 10}{150}} = 1.53, \text{ or, say, } 1\ 9/16 \text{ inch.}$$

Example.—What horsepower would a short shaft, 2 inches in diameter, carrying but two pulleys close to the bearings transmit? The shaft makes 300 revolutions per minute.

$$\text{H.P.} = \frac{2^3 \times 300}{38} = 63.$$

Shafting Subjected to Unusual or Severe Stresses

The formulas given in the preceding paragraphs relate to shafting used under normal conditions. Sometimes the distance between bearings must be abnormally great or the shafts are subjected to very severe stresses between the bearings due to the gears, pulleys, etc., mounted on it. In such cases, it is necessary to calculate the stresses in the shafting and consider both the weight of the shafting itself and

that of the pulleys, gears or other machine parts mounted on it. In calculating the stresses caused by the weight of the shaft itself the total weight beween the bearings is considered as uniformly distributed along the whole shaft, the shaft being considered as a beam freely supported at the bearings. The bending moments caused by pulleys, gears, etc., are then determined and added to find the total bending moment. This, in turn, is then combined with the torsional moment in the manner indicated in the following.

It should be noted that in the case of shafting, the location and direction of the tooth loads, belt pulls, etc., which produce bending, remain fixed while the shaft rotates. The bending stresses are thus constantly varying in direction, and a greater factor of safety should be used than for a beam subjected to a load in one direction only.

Combined Bending and Torsion

If a bar or beam is subjected to both bending and torsion at the same time, the moment to which it is subjected is known as the combined moment and an empirical formula for this condition may be written:

$$\text{Combined moment} = \sqrt{M_b^2 + M_t^2} = SZ$$

in which S = permissible working stress in pounds per square inch;
Z = section modulus for bending (moment of resistance);
M_b = maximum bending moment in inch-pounds;
M_t = maximum torsional moment in inch-pounds.

This formula is entirely empirical. It was published in 1900 by J. J. Guest as the result of experiments made by him. It is apparently not applicable in cases where the torsional moment is very large compared with the bending moment, but for most practical conditions, where it is likely that both moments are of appreciable magnitude, or especially where the bending moment is larger, the formula gives satisfactory values in cases where shafts, beams and machine parts are subjected to combined bending and torsional stresses. The formula has been proved by experiments to be especially applicable to mild steel (machine and structural steel), and as the machine designer is almost exclusively concerned with this material, when dealing with questions of combined bending and twisting stresses, the formula may be accepted as safe and correct for all practical purposes. The safe stress S in this formula may be assumed to be equal to the safe stress in bending.

Example.—Assume that a square bar is subjected to a combined bending and torsional moment. The bending moment is caused by a load of 3000 pounds supported at the outer free end of the bar, the length of which is 30 inches. The torsional moment is 90,000 inch-pounds. Find the size of square bar made from structural steel which will stand the combined moment with a factor of safety of 5.

In this case,

$M_b = 3000 \times 30 = 90{,}000$ inch-pounds;
$M_t = 90{,}000$ inch-pounds;

$S = 60{,}000 \div 5 = 12{,}000$ pounds per square inch;

Z for a square section (see MACHINERY'S HANDBOOK, page 308) $= a^3 \div 6$, in which $a =$ the side of the square.

Then:

$$\text{Combined moment} = \sqrt{90{,}000^2 + 90{,}000^2} = 127{,}000 \text{ approx. and}$$

$$127{,}000 = SZ = 12{,}000 \times \frac{a^3}{6} = 2000\, a^3;$$

$$a^3 = \frac{127{,}000}{2000} = 63.5;$$

$a = 4$ inches, very nearly.

Combined Torsion and Compression

Propeller shafts of steamers and vertical shafts carrying considerable weight, are subjected to combined torsion and compression. Let $P_1 =$ maximum resultant compressive stress; $P_2 =$ maximum resultant shearing stress; $C =$ the compressive stress due to the thrust; $S =$ the shearing stress due to the twisting moment. Then,

$$P_1 = \tfrac{1}{2}\,(C + \sqrt{C^2 + 4S^2}); \qquad P_2 = \tfrac{1}{2}\sqrt{C^2 + 4S^2}$$

It is evident that the safe compressive stress must not exceed P_1, and the safe shearing or torsional stress must not exceed P_2.

Torsional Deflection of Shafting

Shafting must be proportioned not only so that it has the required strength for transmitting a given amount of power, but so that it cannot be twisted through a greater angle than has been found satisfactory by previous experience. Ordinarily, it is assumed by many authorities that the allowable twist in degrees should not exceed five minutes or about 0.08 degree per foot length of the shaft. The following formula gives the angle of torsional deflection of a cylindrical shaft:

$$a = \frac{584\, Tl}{d^4 G}$$

in which $a =$ angle of torsional deflection in degrees;

$T =$ twisting moment in inch-pounds;

$l =$ length of shaft being twisted in inches;

$d =$ diameter of shaft in inches;

$G =$ torsional modulus of elasticity, generally assumed as 12,000,000 for steel shafting.

It will be seen from an inspection of the formula above that in the case of torsional deflection the length of the shaft enters in the calculation. From the previous formulas relating to the strength of the shaft, it will be seen that when mere strength is calculated, the length of the shaft subjected to torsion only does not influence its strength.

Example.—Find the torsional deflection for a shaft 4 inches in diameter and 48 inches long, subjected to a twisting moment of 24,000 inch-pounds.

$$a = \frac{584 \times 24{,}000 \times 48}{4^4 \times 12{,}000{,}000} = 0.22 \text{ degree, or } 13 \text{ minutes.}$$

The deflection per foot, then, equals $13 \div 4 = 3\frac{1}{4}$ minutes, which is within the limits generally allowed.

If it is required to find the diameter of shaft which will give a certain angle of torsional deflection, the following formula may be used:

$$d = \sqrt[4]{\frac{32 \times 12 \times L \times 360 \times PR}{3.14 \times G \times a \times 2 \times 3.14}}$$

in which d = diameter of shaft;

P = force acting on the shaft, producing rotation, in pounds;

R = length of lever arm of force P, in inches;

a = angle of torsional deflection in degrees;

L = length of shaft being twisted in feet;

G = torsional modulus of elasticity (= 12,000,000).

For an angle of deflection equal to 0.08 degree per foot length of the shaft, or a total angle a of $0.08\,L$ degrees,

$$d = 0.29\sqrt[4]{PR} = 4.6\sqrt[4]{\frac{\text{H.P.}}{n}}.$$

Example.—Find the diameter of a lineshaft to transmit 10 horsepower at 150 revolutions per minute, with a torsional deflection not exceeding 0.08 degree per foot of length.

$$d = 4.6\sqrt[4]{\frac{10}{150}} = 2.35 \text{ inches.}$$

It will be seen, by comparing with the section, "Application to Shafting," that a larger diameter is required, in this case, to prevent excessive torsional deflection than is required by mere considerations of strength. For short shafts, it is unnecessary to calculate for the angular deflection. It is only in the case of long shafts that this is necessary, and even then only if the torsional deflection would be objectionable.

Linear Deflection of Shafting

Shafting is subjected to combined bending and twisting moments, the twisting being caused by the forces which give it rotary motion, while the bending is caused partly by the weight of the shaft itself between the bearings and partly by the load placed upon it in the form of pulleys, gears, etc. In the case of shafting, the deflection due to bending must be considered, as well as the torsional deflection and torsional strength. It is considered good practice for line-shafting to limit the deflection to a maximum of 0.010 inch per foot of length. The maximum distance in feet between bearings for average conditions, in order to avoid excessive linear deflection, is determined by the formulas:

$L = \sqrt[3]{720d^2}$ for bare shafts;

$L = \sqrt[3]{140d^2}$ for shafts carrying pulleys, etc.,

in which d = diameter of shaft in inches;

L = maximum distance between bearings in feet.

It is understood that to avoid excessive deflection due to bending, pulleys and gears should be placed as close to the bearings as possible.

Hollow Shafts

For the same weight per linear foot, or, which is the same, for the same area of cross-section or the same amount of material, a hollow shaft is stronger than a solid shaft, because the section modulus for an annular ring is greater than for a "solid" circle *of the same area.* The hollow shaft, of course, has a greater *diameter* than the solid shaft. Calculations will easily show the above statement to be true.

The use of hollow shafts not only reduces the weight of a shaft for a given strength, but increases the reliability of the shafting, on account of the removal of the metal from the core of the shaft. This applies especially to shafts of large diameters, as in large steel ingots the central core is likely to be less dense than the outer portion and to show shrinkage cavities near the center. If the ingot is bored out, the spongy or "piped" portion will be removed, and the metal remaining will be superior in quality to that in a solid shaft. Ingots for shafting should, however, not be cast hollow, but be bored out after having been cast solid.

The following is a simple method given by Mr. E. Hammarstrom in MACHINERY for finding the dimensions of a hollow shaft which can be substituted for a solid shaft of equal strength to resist bending or torsion:

Let D_1 = diameter of solid shaft;
D = outside diameter of hollow shaft;
d = inside diameter of hollow shaft;
$t = \frac{1}{2}(D - d)$ = thickness of metal of hollow shaft;
$k = d \div D$ = ratio of diameters of hollow shaft.

As the hollow shaft is to have the same strength to resist bending as the solid shaft, the moment of resistance of both must be equal. Hence:

$$\frac{\pi(D^4 - d^4)}{32\,D} = \frac{\pi D_1^3}{32}, \text{ from which } D^3 - \frac{d^4}{D} = D_1^3 \qquad (1)$$

If kD is substituted for d in Equation (1):

$$D^3 - D^3k^4 = D_1^3, \text{ from which } \frac{D}{D_1} = \sqrt[3]{\frac{1}{1-k^4}} \qquad (2)$$

In a similar manner, by substituting $\frac{d}{k}$ for D in Equation (1):

$$\frac{d}{D_1} = k\sqrt[3]{\frac{1}{1-k^4}} \qquad (3)$$

Further, as $t = \frac{1}{2}(D - d)$, Formula (4) is found by substitution and simplification:

$$\frac{t}{D_1} = \frac{1-k}{2}\sqrt[3]{\frac{1}{1-k^4}} \qquad (4)$$

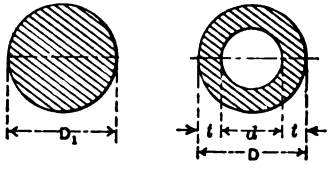
In the accompanying table the values of the factors containing k in

Equations (2), (3), and (4) are calculated for certain values of k. The bottom line of the table gives the weight of the hollow shaft in per cent of that of the solid.

It is evident that Equation (1) would be the same, if it were derived under the assumption that the hollow shaft had the same torsional strength as the solid one, instead of having the same strength against bending, as assumed. The table will therefore hold true for shafts subjected to bending or torsion, or both.

Assume, as an example, that a solid shaft 3 inches in diameter is

TABLE OF FACTORS FOR FINDING DIMENSIONS OF HOLLOW SHAFTS TO REPLACE SOLID SHAFTS



Ratio of	Ratio $d \div D = k$								
	0.50	0 55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
$D \div D_1 =$	1.021	1.032	1.047	1.067	1.095	1.135	1.192	1.279	1.427
$d \div D_1 =$	0.510	0.567	0.628	0.694	0.767	0 851	0.951	1.087	1 284
$t \div D_1 =$	0.257	0.232	0.209	0.186	0.164	0.141	0.119	0.096	0.071
Weight of hollow shaft*....	78.8	74.35	70.2	65.8	61.8	56.4	51.6	45.4	38.7

* Weight of hollow shaft is given in per cent of weight of solid shaft.

to be replaced by a hollow shaft, ratio k being 0.5. Then, by inserting the value found from the table in Equation (2):

$$\frac{D}{D_1} = 1.021 \text{ and } D = 3 \times 1.021 = 3.063 \text{ inches,}$$

$$d = 0.5\,D = 1.532 \text{ inch.}$$

Application to Helical Springs

In helical springs the safe load W that may be placed upon a spring made from a round wire or rod may be found from the following formula:

$$W = \frac{0.4\ Sd^3}{D - d}$$

in which S = safe shearing strength of material in pounds per square inch;

d = diameter of wire or bar from which spring is made;

D = outside diameter of helical spring.

The deflection in *one* coil of a helical spring may be found for a spring made from round wire or rod from the following formula:

$$F = \frac{\frac{1}{2}\ W(D-d)^3}{Gd^4} = \frac{3.14\ S(D-d)^2}{Gd}$$

in which F = deflection in one coil in inches;
W = safe load in pounds;
D = outside diameter of helical spring in inches;
G = torsional modulus of elasticity (12,600,000 for spring steel);
S = safe shearing strength of material in pounds per square inch;
d = diameter of wire or bar from which spring is made, in inches.

Complete formulas covering all classes of springs will be found in standard handbooks. (See MACHINERY'S HANDBOOK, page 412.)

Maximum Safe Stresses in Coil Springs

The following values may be used for the torsional or shearing stresses in coil springs made from a good grade of steel. Assume the ratio of the mean diameter of the spring to the diameter of the bar to equal R; then:

For bars below 3/8 inch diameter:
$R = 3$ $S = 112{,}000$ pounds per square inch.
$R = 8$ $S = 85{,}000$ pounds per square inch.

For bars 7/16 to 3/4 inch in diameter:
$R = 3$ $S = 110{,}000$ pounds per square inch.
$R = 8$ $S = 80{,}000$ pounds per square inch.

For bars from 13/16 to 1 1/4 inch in diameter:
$R = 3$ $S = 105{,}000$ pounds per square inch.
$R = 8$ $S = 75{,}000$ pounds per square inch.

For bars over 1¼ inch in diameter a stress of more than 100,000 should not be used. Where a spring is subjected to sudden shocks, a smaller value of S is necessary.

These values are applicable to compression springs with open coils. Experience has shown that in close-coiled springs and extension springs the safe value of the stress per square inch, S, is only about two-thirds of that for open-coiled compression springs of the same dimensions. The safe torsional or shearing strength for spring brass and phosphor-bronze may be taken as 25,000 pounds per square inch. The torsional modulus of elasticity may be taken as 6,000,000 for spring brass and phosphor-bronze, and 12,600,000 for steel.

The best proportions for coil springs is to use an outside diameter of the spring equal to from five to eight times the diameter of wire or bar from which the spring is made; under no circumstances should the outside diameter be made less than four times the diameter of the wire. The effective number of coils in a compression spring may be considered as 2 less than the actual number, owing to the squared ends of the spring. Springs of small diameter may be safely subjected to a higher unit stress than those of large diameter.

Materials Used for Springs

Steel containing about one per cent carbon and comparatively free from phosphorus and sulphur, generally known as spring steel, is ordinarily used for springs.

For small springs, music wire is used to a great extent and is the best material obtainable for this purpose. It is especially recommended for devices where the spring is compressed frequently and suddenly. Vanadium steel has recently come into use to a considerable extent for springs. The addition of a small percentage of vanadium to steel increases the elasticity of the material, but the cost of springs made from this material is considerably higher.

Brass and phosphor-bronze should be used for springs that must resist moisture. These springs, however, are much more expensive than steel springs, both on account of the higher cost of the material, and because the permissible stress is less, thus making larger sizes of these springs necessary for the same capacity.

Factor of Safety in Springs Frequently Compressed

When a spring acts only occasionally it can be safely designed to carry a load which causes a fiber stress nearly equal to the elastic limit of the spring, but when the compressions or extensions are frequent, a larger factor of safety must be used. A valve spring in an automobile motor, for example, which operates, say, 200 times a minute, should have a factor of safety of at least 4. In other words, a spring made of ⅛-inch wire, which ordinarily could be designed for a torsional stress of 100,000 pounds per square inch, should be designed to work at a stress not over 25,000 pounds per square inch when used in service of the kind mentioned.

High-class springs, such as valve springs, should have the ends squared and ground at right angles to their axis; the outside diameter should be at least one-third of the length, and it should be supported its entire length, unless it is very short, in order to prevent buckling, which introduces bending and twisting strains. High-class valve springs when placed on end on a flat plate should not vary more than ½ degree from the perpendicular to the plate. These springs should be protected from rusting by a good coat of japan, baked on, or by electro-galvanizing.

CHAPTER V

MISCELLANEOUS APPLICATIONS

Strength of Columns or Struts

When a member subjected to compression stresses has considerable length in proportion to its width, depth or diameter, the ordinary formulas for compression are not applicable, because bending stresses are set up on account of the length of the column or strut, and these stresses tend to bend or bulge the member. A number of empirical formulas have been devised for calculating the strength of columns. These formulas are all based upon what is known as the Gordon formula. In the formulas given in the following,

p = ultimate load in pounds per square inch;

l = length of column or strut in inches;

r = radius of gyration in inches.

The radius of gyration r is found from the moment of inertia and the area of section as follows:

$$r = \sqrt{\frac{\text{moment of inertia}}{\text{area of section}}}$$

To find the safe load for a given section from the formulas given in the following, it is necessary to multiply the value of p, as found from the formulas, by the area of the section, thus finding the total ultimate load for the whole section; then divide this load by the factor of safety to find the safe load that may be placed on the column or strut. Formulas for seven different cases will be given.

1. Assume that a steel column has both ends fixed or resting on a flat support, preventing any sidewise motion. The formula for this case is:

$$p = \frac{50{,}000}{1 + \frac{l^2}{36{,}000\, r^2}}$$

2. For a steel column with one end fixed or resting on a flat support and with the other end round or hinged, the formula is:

$$p = \frac{50{,}000}{1 + \frac{l^2}{24{,}000\, r^2}}$$

3. For a steel column with both ends round or hinged, the formula is:

$$p = \frac{50{,}000}{1 + \frac{l^2}{18{,}000\, r^2}}$$

4. For a round cast-iron column, solid, having both ends fixed or resting on a flat support, and where d is the diameter of the column, the formula is:

$$p = \frac{80{,}000}{1 + \frac{l^2}{800\,d^2}}$$

5. For a cast-iron column, circular in cross-section, but cast hollow and having both ends fixed or resting on flat supports, the formula is (d = outside diameter of column):

$$p = \frac{80{,}000}{1 + \frac{l^2}{800\,d^2}}$$

6. For a cast-iron column, square cross-section, cast with a hollow square in the center and having both ends fixed or resting on flat supports, the formula is (s = outside dimension of square):

$$p = \frac{80{,}000}{1 + \frac{l^2}{1000\,s^2}}$$

7. For a square wooden column with flat supports or with both ends fixed, in which the side of the square is s, the formula is:

$$p = \frac{5000}{1 + \frac{l^2}{250\,s^2}}$$

Example:—What would be the load that could safely be carried by a steel column of bar stock, 2 inches in diameter, 5 feet long, with a factor of safety of 4, assuming that both ends of the bar are hinged?

$$p = \frac{50{,}000}{1 + \frac{(5 \times 12)^2}{18{,}000 \times (0.5)^2}} = \frac{50{,}000}{1 + 0.8} = 27{,}800 \text{ pounds per square inch.}$$

Safe load on column:

$$\frac{27{,}800 \times 3.14}{4} = 21{,}800 \text{ pounds.}$$

Pipe Columns

The allowable compressive stress for steel pipe columns may be determined from the formula:

$$S = 15{,}200 - 58\,L \div R$$

in which S = allowable compressive stress in pounds per square inch; L = length of column in inches; R = radius of gyration in inches. This formula is applicable to steel pipe columns with flat ends. No columns should be used having an unsupported length greater than 120 times its radius of gyration. The formula is based upon the requirements of the New York Building Code.

A similar formula, based upon the Chicago Building Ordinances, is:

$$S = 16{,}000 - 70\,L \div R$$

in which the letters denote the same quantities as in the previous formula.

Flat Stayed Surfaces

In many cases, large flat areas are held against pressure by stays distributed at regular intervals over the surface. In boiler work, these stays are usually screwed into the plate and the projecting end riveted over to insure steam tightness. The U. S. Board of Supervising Inspectors and the American Boiler Makers' Association rules give the following formula for flat stayed surfaces:

$$P = \frac{C \times t^2}{S^2}$$

in which P = pressure in pounds per square inch;

C = a constant which equals 112, for plate 7/16 inch and under; 120, for plates over 7/16 inch thick; 140, for plates with stays having a nut and bolt on the inside and outside; and 160, for plates with stays having washers of at least one-half the thickness of the plate, and with a diameter at least one-half of the greatest pitch.

t = thickness of plate in sixteenths of an inch (thickness = 7/16, $t = 7$);

S = greatest pitch of stays in inches.

Strength of Flat Plates

The machine designer is often called upon to carry out designs consisting in part of flat surfaces, such as plates supported or fixed at the edges, with or without intermediate supports or ribs. Exact formulas for finding the bending moments of flat plates supported along their edges and subjected to stresses created by pressures normal to their surfaces have not been determined. The formulas given by different authorities are founded on assumptions and should be considered as approximations only; they should be used with caution, as the results obtained are not likely to be very accurate.

A square cast-iron plate rigidly held at the edges and loaded with a uniformly distributed load, or a load concentrated at the center, would be likely to fail as shown in Fig. 29. It would first fracture along the diagonal lines from A to B and then fail at or near the fixed edges along lines BB. The plate might also shear off along the edges BB, depending upon the method of loading and the thickness of the plate. If the plate were merely supported along all the four edges, but not rigidly held, it would be likely to fail by breaking along the diagonal lines AB only.

In Fig. 30 is illustrated the probable manner of failure of a flat rectangular plate of cast iron, loaded with a uniformly distributed load. The plate, if secured along all the four edges, would probably fail by fracturing along the center line AA of the long axis of the

plate and along the diagonal lines *AB*, and then fail at or near the edges of the support along the lines *BB*. If the plate were merely supported along all four edges, it would fail simply by fracturing along the center line *AA* and the diagonal lines *AB*. A plate firmly secured at the edges offers greater resistance to the stress created by the load than does a plate merely supported at the edges.

Approximate formulas for round, square and rectangular plates will be found in standard handbooks. An unusually complete set of formulas will be found in MACHINERY'S Data Book No. 17, "Mechanics and Strength of Materials," and also in MACHINERY'S HANDBOOK, page 363. While the formulas given are approximate only, it is important that formulas be deduced and used for designs of this character,

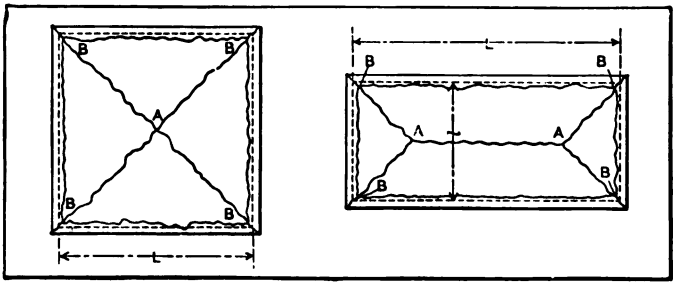


Fig. 29 Fig. 30

because they indicate, in a general way, the dimensions required, and the factor of safety assumed will always be taken large enough so that, practically, the approximate nature of the formulas does not detract from their value.

Spherical Shells Subjected to Internal Pressure

In the following formulas for spherical shells subjected to internal pressure let,

D = internal diameter of shell in inches;
P = internal pressure in pounds per square inch;
S = safe tensile stress per square inch;
t = the thickness of metal in the shell in inches. Then:

$$P\frac{\pi D^2}{4} = \pi DtS, \text{ and } t = \frac{PD}{4S}$$

This formula also applies to hemi-spherical shells, such as the hemispherical head of a cylindrical container subjected to internal pressure, etc.

Example.—Find the thickness of metal required in the hemispherical end of a cylindrical vessel, 2 feet in diameter, subjected to an internal pressure of 500 pounds per square inch. The material is mild steel and a tensile stress of 10,000 pounds per square inch is allowable.

$$t = \frac{500 \times 2 \times 12}{4 \times 10{,}000} = 0.3 \text{ inch.}$$

If the radius of curvature of the dome head of a boiler or container subjected to internal pressure is made equal to the diameter of the boiler, the thickness of the cylindrical shell and of the spherical head should be made the same. For example, if a boiler is 3 feet in diameter, the radius of curvature of its head should be made 3 feet, if material of the same thickness is to be used and the stresses are to be equal in both the head and cylindrical portion.

Collapsing Pressures of Cylinders and Tubes Subjected to External Pressures

The following formulas may be used for finding the collapsing pressures of modern lap-welded Bessemer steel tubes:

$$P = 86{,}670 \frac{t}{D} - 1386 \qquad (1)$$

$$P = 50{,}210{,}000 \left(\frac{t}{D}\right)^3 \qquad (2)$$

in which P = collapsing pressure in pounds per square inch; D = outside diameter of tube or cylinder in inches; t = thickness of wall in inches.

Formula (1) is for values of P greater than 580 pounds per square inch, and Formula (2) is for values of P less than 580 pounds per square inch. These formulas are substantially correct for all lengths of pipe greater than six diameters between transverse joints that tend to hold the pipe to a circular form. The pressure P found is the actual collapsing pressure, and a suitable factor of safety must be used. Ordinarily, a factor of safety of 5 is sufficient. In cases where there are repeated fluctuations of the pressure, vibration, shocks and other stresses, a factor of safety of from 6 to 12 should be used.

The Formulas (1) and (2), for steel tubes, given above, were determined by Prof. R. T. Stewart, Dean of the Mechanical Engineering Department of the University of Pittsburg, in a series of experiments carried out at the plant of the National Tube Co., McKeesport, Pa. These tests occupied a period of four years. A full report of the details of these experiments will be found in a paper presented by Prof. Stewart before the American Society of Mechanical Engineers in May, 1906. The principal conclusions to be drawn from the results of this research may be briefly stated as follows:

The length of tube, between transverse joints tending to hold it to a circular form, has no practical influence upon the collapsing pressure of a commercial lap-welded steel tube, so long as this length is not less than about six times the diameter of the tube.

The apparent fiber stress under which the different tubes failed varied from about 7000 pounds per square inch for the relatively thinnest to 35,000 pounds per square inch for the relatively thickest walls. Since the average yield point of the material tested was 37,000

pounds and the tensile strength 58,000 pounds per square inch, it is evident that the strength of a tube subjected to external fluid collapsing pressure is not dependent alone upon the elastic limit or ultimate strength of the material from which it is made.

Strength of Cylinders Subjected to Internal Pressure

In low-pressure work, the general practice is to make the thickness of the metal equal to the internal diameter in inches times the pressure in pounds per square inch, and this product divided by twice the allowable working stress of the material. To this is added a variable quantity to allow for unsound castings and possible unknown stresses. Hence, if t = thickness in inches; d = inside diameter in inches; P = pressure in pounds per square inch; S = allowable tensile stress in pounds per square inch, then:

$$t = \frac{dP}{2S}.$$

To the value of t thus obtained must then be added an amount to allow for variations in the material and possible excessive stresses when the cylinder is in operation.

Example.—Find the thickness required for a cast-iron cylinder, 15 inches in diameter (inside), to withstand an internal pressure of 200 pounds per square inch. Assume the allowable working stress for cast iron to be 4000 pounds per square inch. Then:

$$t = \frac{15 \times 200}{2 \times 4000} = \frac{3}{8} \text{ inch.}$$

The material being cast iron, a liberal allowance must be added to this thickness to take care of possible defects in the casting.

The formula given should be used only for low pressures. When the pressures rise, the Barlow formula is preferable. This formula is similar in form to the one already given, but it gives results quite different when applied to tubes and pipes having walls of considerable thickness in proportion to the diameter, because the Barlow formula is expressed in terms of the outside diameter, whereas the formula given above is expressed in terms of the inside diameter. The Barlow formula is:

$$t = \frac{DP}{2S}$$

in which t = thickness in inches; D = outside diameter in inches; P = pressure in pounds per square inch; S = allowable tensile stress in pounds per square inch.

This formula is based on assumptions which cannot be considered as theoretically correct, but the error is on the side of safety, and experiments have proved that of the various formulas proposed for the strength of tubes and pipes subjected to moderate pressures, the Barlow formula gives the most reliable results.

The average ultimate tensile strength of seamless steel tubes may be assumed at 55,000 pounds per square inch; that for butt-welded steel

pipe at 40,000; that for lap-welded steel pipe at 50,000; and that for wrought-iron pipe (butt-welded or lap-welded) at 28,000 pounds per square inch.

If seamless steel tubes are assumed to have a strength of 100 per cent, butt-welded steel pipe has a comparative strength of 73 per cent, and lap-welded steel pipe of 92 per cent. From this it will be seen that the strength of a butt-weld is only about 80 per cent of that of a lap-weld. The relative strengths of wrought-iron and steel pipe are as follows: Butt-welded wrought-iron pipe has 70 per cent of the strength of similar butt-welded steel pipe, and lap-welded wrought-iron pipe has 57 per cent of the strength of similar lap-welded steel pipe.

Cylinders Subjected to High Internal Pressure

For high pressures, Lamé's formula is used. This formula is in its usual form,

$$t = r\left(\sqrt{\frac{S+P}{S-P}} - 1\right)$$

sometimes inconvenient to use. The following forms of the same formula obtained by substitution are often useful:

$$S = P\frac{R^2 + r^2}{R^2 - r^2} \qquad R = r\sqrt{\frac{S+P}{S-P}}$$

$$P = S\frac{R^2 - r^2}{R^2 + r^2} \qquad r = R\sqrt{\frac{S-P}{S+P}}$$

In these formulas:

S = maximum allowable fiber stress per square inch;
R = outer radius of cylinder in inches;
r = inner radius of cylinder in inches;
P = pressure within the cylinder in pounds per square inch;
$t = R - r$ = thickness of cylinder in inches.

Unless very high-grade material is used and sound castings assured, cast iron should not be used for pressures exceeding 2000 pounds per square inch. When pressures exceed 2500 pounds per square inch, the packings are likely to leak and the valves and pipe fittings give trouble. It is, therefore, advisable to keep the pressure below this point, if possible. It is well to leave more metal in the bottom of a hydraulic cylinder than is indicated by the results of calculations, because a hole of some size must be cored in the bottom to permit the entrance of a boring bar when finishing the cylinder, and when this hole is subsequently tapped and plugged it often gives trouble if the precaution mentioned is not taken.

For steady or gradually applied stresses, the maximum allowable fiber stress S in the formulas above may be assumed from 3500 to 4000 pounds per square inch for cast iron; from 6000 to 7000 pounds per square inch for brass; and as 12,000 pounds per square inch for steel castings. For intermittent stresses, such as in cylinders for steam and hydraulic work, 3000 pounds per square inch is the

maximum that can be used with ample safety for cast iron; 5000 pounds per square inch for brass; and 10,000 pounds per square inch for steel castings.

Example: Find the thickness of a cast iron cylinder to withstand a pressure of 1,000 pounds per square inch; the inside diameter of the cylinder is to be 10 inches, and the maximum allowable fiber stress per square inch 4,000 pounds.

The thickness is found by the following formula:

$$t = r\left(\sqrt{\frac{S+P}{S-P}} - 1\right)$$

in which t = thickness of cylinder wall in inches,

r = inside radius of cylinder in inches,

P = working pressure in pounds per square inch,

S = allowable fiber stress in pounds per square inch.

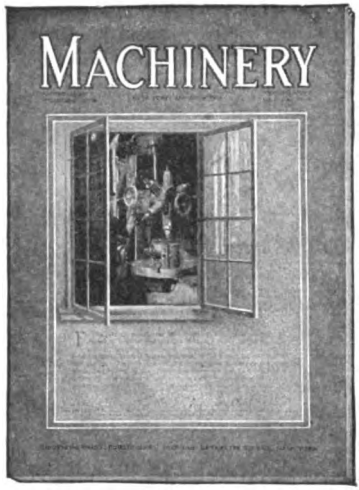
Inserting the given values, we have:

$$t = \frac{10}{2}\left(\sqrt{\frac{4000+1000}{4000-1000}} - 1\right) = 5\,(\sqrt{1.667} - 1) =$$

$$5\,(1.29 - 1) = 5 \times 0.29 = 1.45,\ \text{or say } 1\tfrac{1}{2} \text{ inch.}$$

Conclusion

As indicated by the directions and formulas given in the preceding pages, accurate calculations can be made for the strength of a great many of the component parts that enter into machine design; but it remains a fact nevertheless, that there are many instances where calculations are impossible, or where they would be so involved, cumbersome or uncertain, that judgment and experience are almost the only guides upon which the machine designer can depend. Wherever calculations can be conveniently made, they should be carried out; but past successful designs and the results of practical experience should be compared with the calculated results whenever possible, as in this way theory and practice can be made to supplement each other in a most valuable way. There are many practical conditions, especially when dealing with such metals as cast-iron, which require consideration, in addition to the mere calculated strength; and in all cases where a machine part is exposed to excessive wear, this factor must be considered also. Sometimes, as in the case of gear teeth, wear may be a more important factor than strength, and in such cases the designer must necessarily base the dimensions necessary upon the results of designs that have worked successfully in the past. There are no hard and fast rules in regards to this problem of machine design, and judgment and experience will always prove most valuable aids to the designer, no matter how well versed he may be in the theory of the strength of materials.



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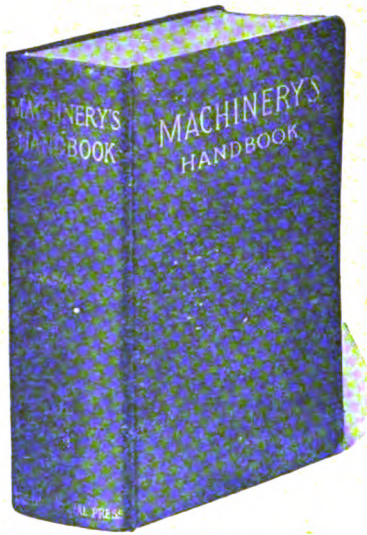

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